UNCLASSIFIED

AD NUMBER	
AD005479	
CLASSIFICATION CHANGES	
TO:	unclassified
FROM:	confidential
LIMITATION CHANGES	
TO: Approved for public release; distribution is unlimited.	
FROM: Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; JAN 1953. Other requests shall be referred to Army Ballistic Research Laboratory, Aberdeen Proving Ground, MD 21005.	
AUTHORITY	
31 jan 1965, DoDD 5200.10; brl ltr, 22 apr 1981	

THIS PAGE IS UNCLASSIFIED

BRL

REPORT No. 843

On the Effect of Thickness on the Damping in Roll of Airfoils at Supersonic Speeds

JOHN C. MARTIN
and
NATHAN GERBER

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

REPORT NO. 843

January 1953

ON THE EFFECT OF THICKNESS ON THE DAMPING IN ROLL OF AIRFOILS AT SUPERSONIC SPEEDS

by

John C. Martin

and

Nathan Gerber

Project No. TB3-0108H of the Research and Development Division, Ordnance Corps

A B E R D E E N   P R O V I N G   G R O U N D,   M A R Y L A N D

BALLISTIC RESEARCH LABORATORIES

REPORT NO. 843

JCMartin/NGerber/mer
Aberdeen Proving Ground, Md.
January 1953

ON THE EFFECT OF THICKNESS ON THE DAMPING IN ROLL OF AIRFOILS AT SUPERSONIC SPEEDS

ABSTRACT

The effect of thickness on the damping in roll of airfoils at supersonic speeds is investigated. The C_{l_p} and the lifting pressure distribution for several simple airfoils are found by the use of a second order theory similar to the second order theory introduced by Busemann and extended by Van Dyke.

The airfoils considered consist of infinite wings with arbitrary symmetrical cross sections both swept and unswept, and of a reversed delta wing which has a parabolic cross section.

The flows considered are three-dimensional flows, and the methods employed are somewhat different from those used by Van Dyke. The possibility of using the methods employed in the present paper for other flow problems is discussed briefly.

SYMBOLS

AR	aspect ratio
a	local velocity of sound
b	airfoil span
c	velocity of sound in free stream
c_o	velocity of sound in air which has been brought to rest adiabatically
C_p	pressure coefficient $\left(\frac{\text{Pressure}}{1/2\ \rho V^2}\right)$
C_{l_p}	damping in roll $\lim_{p \to o} \left(\frac{\text{rolling moment}}{1/2\ \rho\ V^2\ \frac{pb}{2V}\ Sb}\right)$
d	airfoil chord
f	arbitrary function associated with the equation of the airfoil surface (see equation (59))
k, k_1, k_2, k_3	constants
K(t)	arbitrary function of time
l	a distance small compared to unity
M	Mach number
m	slope of leading edge (cot Λ)
N =	$\frac{(\gamma + 1)\ M^2}{2\beta^2}$
$\bar{n}_u$ =	$-i\beta^2\ \nu_1 + j\nu_2 + k\nu_3$
P	pressure
p	rate of roll
q	velocity vector

$R =$	$\sqrt{(x-\xi)^2 - \beta^2(y-\eta)^2 - \beta^2(z-\zeta)^2}$
r, θ	polar coordinates
S	area of airfoil
s	equation of airfoil surface
t	time
V	free stream velocity
x, y, z	rectangular coordinates
$\alpha =$	$\sqrt{\beta^2 m^2 - 1}$
$\beta =$	$\sqrt{M^2 - 1}$
$\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4$	auxiliary functions used in finding the second order potential functions
γ	adiabatic exponent
ϵ	parameter small compared to unity
ξ, η, ζ	rectangular coordinates
$\theta_1\ \theta_2$, and θ_3	auxiliary functions used in finding the second order potential function
Λ	leading edge sweep back angle
λ	minus the slope of the center line of a reversed delta wing
ν_1, ν_2, ν_3	direction cosines of outward normal to element of area da
ρ	density
$\sigma_x =$	$\epsilon \frac{\partial f}{\partial x}$
$\sigma_y =$	$\epsilon \frac{\partial f}{\partial y}$
Φ	second order perturbation potential function
ϕ	first order perturbation potential
ψ	second order correction potential
Ω	scalar potential function

INTRODUCTION

The damping in roll coefficient, C_{l_p}, provided by fins and wings is important in the design of certain types of finned missiles and in stability studies of aircraft. Fortunately for many cases the effects of missile bodies and aircraft fuselages on the C_{l_p} can be neglected. In these cases the C_{l_p} can be found from a study of the isolated airfoils. The use of the linearized theory of supersonic flow has permitted an evaluation of the C_{l_p} for a number of different types of wings and fins (for example see references 1 through 6). Expressions for the C_{l_p} found by the linearized theory depend upon the Mach number and the planform geometry and are independent of the thickness distribution. Tests conducted in the free flight Aerodynamic Range of the Ballistic Research Laboratories at Aberdeen Proving Ground (reference 8) indicate, however, that for certain rectangular fins the thickness distribution has an appreciable effect on the C_{l_p}. Thus, it appears that a study of the second order flow around rolling airfoils might yield some useful information.

The present paper presents the C_{l_p} for several simple airfoils calculated on the basis of a second order theory similar to the one introduced by Busemann (reference 9) and extended by Van Dyke (references 10 and 11). The airfoils considered herein consist of infinite wings with arbitrary symmetrical cross sections both swept and unswept, and of a reversed delta wing which has a parabolic cross section. It is the authors' opinion that from the study of these relatively simple airfoils the effect of thickness on many other airfoils can be estimated.

The analysis contained in the present paper utilizes the same type of iteration as that used by Van Dyke in reference (10 and 11). There are, however, two distinct differences between the analyses of Van Dyke's and the present paper. Firstly, the partial differential equations differ; secondly, the methods of determining the second order solution are not the same.

ANALYSIS

Introduction:

Recent work by Milton D. Van Dyke (references 10 and 11) indicates that second order solutions of the partial differential equation of steady supersonic flow can be obtained by the use of iterative methods.

The partial differential equation considered here is not the equation of steady supersonic flow. It is quite similar, however, and we shall assume that the second order solution can be obtained by iterative methods.

It will also be assumed that the characteristics are the same for the first and second order solutions (this assumption was made in references 10 and 11). For steady plane flow the second order solution (reference 10) found by using an iterative method based on the preceding assumption yields the correct second order pressure of the Busemann second order theory. Unfortunately, no such justification of the preceding assumption is known to the authors for three dimensional flows.

The iterative method used in references 10 and 11 requires a particular integral of the nonhomogeneous second order partial differential equation in terms of the first order solution. The particular integrals utilized in the present paper apply only to the particular problem under consideration. The method developed here has the advantage that it can be applied to problems in three dimensional flow where a particular integral of the nonhomogeneous second order equation in terms of the first order solution is not available in the literature.

The Partial Differential Equation:

The partial differential equation for the potential function to be used is a special case of the three dimensional time dependent equation for the potential function of a non-viscous compressible fluid. For completeness this equation will be determined from the equations of hydrodynamics.

Four of the equations needed are contained in the equation of continuity:

$$\nabla \cdot \rho q + \frac{\partial \rho}{\partial t} = 0 \tag{1}$$

and in the three Euler equations:

$$\frac{1}{\rho} \nabla P + \frac{1}{2} \nabla q^2 + \frac{\partial q}{\partial t} = 0 \tag{2}$$

where q is the velocity vector and ρ is the density.

Equations (1) and (2) give four relations between the five variables P, ρ and the three components of q. A fifth relation is obtained from the assumption of zero heat transfer (constant entropy). In the case of a perfect gas this relation is

$$P = k \rho^{\gamma} \tag{3}$$

where k is a constant and γ is the adiabatic exponent.

It will be assumed that the velocity can be expressed as the gradient of a potential function;

$$q = \nabla \Omega \tag{4}$$

Equation (2) may therefore be written as

$$\frac{1}{\rho} \nabla P + \frac{1}{2} \nabla q^2 + \frac{\partial}{\partial t} \nabla \Omega = 0$$

The preceding equation can be integrated to yield

$$\Omega_t + \frac{1}{2} q^2 + \int \frac{dP}{\rho} = K(t)$$

where K(t) is an arbitrary function of time. The function K(t) is a constant (to be denoted by K) for problems which will be considered here since the flow upstream of the airfoils is independent of time.

The result of evaluating the integral in the preceding equation by the use of equation (3) is

$$\Omega_t + \frac{1}{2} q^2 + \frac{\gamma k}{(\gamma-1)} \rho^{\gamma-1} = K \tag{5}$$

The first partial derivative of equation (5) with respect to time can be expressed as (with the aid of equation (3))

$$\Omega_{tt} + \frac{1}{2} \frac{\partial q^2}{\partial t} + \frac{1}{\rho} \frac{\partial P}{\partial t} = 0 \tag{6}$$

Equation (1) can be expressed as

$$\nabla \cdot q + \frac{1}{\rho} (q \cdot \nabla \rho + \frac{\partial \rho}{\partial t}) = 0 \tag{7}$$

Denoting $\frac{dP}{d\rho}$, the square of the velocity of sound, by a^2 it follows that equation (7) can be written as

$$\nabla \cdot q + \frac{1}{a^2} (q \cdot \frac{1}{\rho} \nabla P + \frac{1}{\rho} \frac{\partial P}{\partial t}) = 0 \tag{8}$$

Eliminating P and ρ from equation (8) by the use of equations (2) and (6) yields

$$a^2 \nabla \cdot q = \Omega_{tt} + q \cdot \nabla \frac{q^2}{2} + \frac{\partial}{\partial t} q^2 \tag{9}$$

Equation (9) can be written in terms of the derivatives of the potential function and a^2 as

$$a^2 \nabla^2 \Omega = \Omega_{tt} + \frac{1}{2} \nabla \Omega \cdot \nabla (\nabla\Omega \cdot \nabla\Omega) + \frac{\partial}{\partial t} (\nabla\Omega \cdot \nabla\Omega) \tag{10}$$

Equation (10) is the three dimensional time dependent partial differential equation for the potential function provided a^2 is expressed in terms of the derivatives of the potential function.

By definition

$$\frac{dP}{d\rho} = a^2$$

By equation (3)

$$a^2 = k \gamma \rho^{\gamma - 1}$$

Substituting this relation into equation (5) and solving for a^2 yields

$$a^2 = c_o^2 - \frac{\gamma-1}{2} (\Omega_x^2 + \Omega_y^2 + \Omega_z^2 + 2 \Omega_t) \tag{11}$$

The function a is the local velocity of sound and the constant c_o is the velocity of sound in the gas which has been brought to rest adiabatically. It is convenient to express a^2 in terms of c, the velocity of sound in the undisturbed stream ahead of ~~the airfoil~~. It follows from equation (11) that c is

$$c^2 = c_o^2 - \left(\frac{\gamma-1}{2}\right) V^2$$

The result of eliminating c_o^2 from the preceding equation and equation (11) is

$$a^2 = c^2 - \frac{\gamma-1}{2} (\Omega_x^2 + \Omega_y^2 + \Omega_z^2 + 2 \Omega_t - V^2) \tag{12}$$

A perturbation potential will be used which is defined by

$$\Omega = V (x + \Phi)$$

Note that the perturbation potential has been normalized through division by the free stream velocity.

It will be assumed that for the airfoils considered here the triple products in equation (15) can be neglected. Based on this assumption equation (15) reduces to

$$\underline{-\beta^2 \Phi_{xx} + \Phi_{yy} + \Phi_{zz} - \frac{2V}{c^2} \Phi_{xt} - \frac{1}{c^2} \Phi_{tt}} = M^2 \Big\{ (\gamma-1) \Big(\Phi_x +$$

$$\frac{1}{V} \Phi_t \Big) \Big(\Phi_{xx} + \Phi_{yy} + \Phi_{zz} \Big) + 2 \Phi_x \Phi_{xx} + 2 \Phi_y \Phi_{xy} + \qquad (16)$$

$$2 \Phi_z \Phi_{xz} + \frac{2}{V} \Phi_x \Phi_{xt} + \frac{2}{V} \Phi_y \Phi_{yt} + \frac{2}{V} \Phi_z \Phi_{zt} \Big\}$$

Equation (16) can be expressed in cylindrical coordinates by

$$\underline{-\beta^2 \Phi_{x'x'} + \Phi_{r'r'} + \frac{1}{r'} \Phi_{r'} + \frac{1}{(r')^2} \Phi_{\theta'\theta'} - \frac{2V}{c^2} \Phi_{x't'} - \frac{1}{c^2} \Phi_{t't'}} =$$

$$M^2 \Big\{ (\gamma-1) \Big(\Phi_{x'} + \frac{1}{V} \Phi_{t'} \Big) \Big(\Phi_{x'x'} + \Phi_{r'r'} + \frac{1}{r'} \Phi_{r'} + \frac{1}{(r')^2} \Phi_{\theta'\theta'} \Big) +$$

$$\qquad (17)$$

$$2 \Phi_{x'} \Phi_{x'x'} + \frac{2}{(r')^2} \Phi_{\theta'} \Phi_{x'\theta'} + 2 \Phi_{r'} \Phi_{x'r'} + \frac{2}{V} \Phi_{x'} \Phi_{x't'} +$$

$$\frac{2}{V(r')^2} \Phi_{\theta'} \Phi_{\theta't'} + \frac{2}{V} \Phi_{r'} \Phi_{r't'} \Big\}$$

The coordinate axes of equation (17) are fixed in space. It is more convenient, however, to choose a new set of axes fixed to and rolling with the body, the x-axis coinciding with the axis of roll. The relations between the components of the two sets of axes are:

$$x = x'$$
$$r = r'$$
$$\theta = \theta' - pt'$$
$$t = t'$$

In the new coordinates equation (17) becomes

$$-\beta^2 \Phi_{xx} + \Phi_{rr} + \frac{1}{r} \Phi_r + \frac{1}{r^2} \Phi_{\theta\theta} - \frac{2V}{c^2} \Phi_{xt} + \frac{2Vp}{c^2} \Phi_{x\theta} - \frac{1}{c^2} \Phi_{tt} + \frac{2p}{c^2} \Phi_{tt} - \frac{p^2}{c^2} \Phi_{\theta\theta} = M^2 \Big\{ (\gamma-1) \left(\Phi_x + \frac{1}{V} \Phi_t - \frac{p}{V} \Phi_\theta \right) \cdot$$

$$\cdot \left(\Phi_{xx} + \Phi_{rr} + \frac{1}{r} \Phi_r + \frac{1}{r^2} \Phi_{\theta\theta} \right) + 2 \Phi_x \Phi_{xx} +$$

$$\frac{2}{r^2} \Phi_\theta \Phi_{x\theta} + 2 \Phi_r \Phi_{xr} - \frac{2p}{V} \Phi_x \Phi_{x\theta} - \frac{2p}{Vr^2} \Phi_\theta \Phi_{\theta\theta} +$$

$$- \frac{2p}{V} \Phi_r \Phi_{r\theta} + \frac{2}{V} \Phi_x \Phi_{xt} + \frac{2}{Vr^2} \Phi_\theta \Phi_{\theta t} + \frac{2}{V} \Phi_r \Phi_{rt} \Big\}$$

The flow over bodies rolling with constant velocities is independent of time; thus, for the cases to be considered here the time derivatives are zero in the preceding equation. Terms such as

$-\frac{p^2}{c^2} \Phi_{\theta\theta}$, $-\frac{2p}{r} \Phi_x \Phi_{x\theta}$, etc are of the third order and can

be neglected. It follows from the preceding equation that the partial differential equation (correct to the second order) for the flow over bodies with constant rates of roll is given (in rectangular coordinates) by

$$-\beta^2 \Phi_{xx} + \Phi_{yy} + \Phi_{zz} = -\frac{2Vp}{c^2} (y \Phi_{xz} - z \Phi_{xy}) +$$

$$M^2 \Big\{ (\gamma-1) \Phi_x \left(\Phi_{xx} + \Phi_{yy} + \Phi_{zz} \right) + 2 \Phi_x \Phi_{xx} + \tag{18}$$

$$2 \Phi_y \Phi_{xy} + 2 \Phi_z \Phi_{xz} \Big\}$$

Equation (18) will be used in the following analysis of rolling airfoils.

Pressure Relation:

The pressure coefficient to be used is a special case of the pressure coefficient for a three dimensional time dependent flow.

The pressure coefficient will be defined by

$$C_p = \frac{P-P_o}{\frac{1}{2}\rho_o V^2} = \frac{2}{V^2}\left(\frac{P}{\rho_o} - \frac{P_o}{\rho_o}\right) \tag{19}$$

The square of the local velocity of sound is

$$a^2 = \frac{dP}{d\rho}$$

From this and equation (3) it follows that

$$a^2 = \frac{dP}{d\rho} = \gamma\, k\, \rho^{\gamma-1} = k^{\frac{1}{\gamma}}\, \gamma\, P^{\frac{\gamma-1}{\gamma}}$$

and (20)

$$\frac{P}{\rho} = k\, \rho^{\gamma-1} = \frac{a^2}{\gamma}$$

In the undisturbed stream

$$\frac{P_o}{\rho_o} = \frac{c^2}{\gamma}$$

The pressure coefficient can now be written as

$$C_p = \frac{2}{\gamma M^2}\left(\frac{P}{P_o} - 1\right) \tag{21}$$

From equations (20) the pressure can be written as

$$P = \left(\frac{a^2}{\gamma k^{1/\gamma}}\right)^{\frac{\gamma}{\gamma-1}}$$

In the undisturbed stream

$$Po = \left(\frac{c^2}{\gamma k^{1/\gamma}}\right)^{\frac{\gamma}{\gamma-1}}$$

From the two preceding relations

$$\frac{p}{p_o} = \left(\frac{a^2}{c^2}\right)^{\frac{\gamma}{\gamma-1}}$$

Substituting this expression into equation (21) yields

$$C_p = \frac{2}{\gamma M^2}\left[\left(\frac{a^2}{c^2}\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$

Substituting equation (14) into the preceding equations gives

$$C_p = \frac{2}{\gamma M^2}\left\{\left[1 - \frac{\gamma-1}{2} M^2 \left(2\Phi_x + \Phi_x^2 + \Phi_y^2 + \Phi_z^2 + \frac{2}{V}\Phi_t\right)\right]^{\frac{\gamma}{\gamma-1}} - 1\right\} \tag{22}$$

Equation (22) is the pressure coefficient for a three dimensional unsteady flow.

Expanding equation (22) about the free stream conditions yields

$$C_p = -2\left(\Phi_x + \frac{\Phi_t}{V}\right) - \Phi_y^2 - \Phi_z^2 + \beta^2 \Phi_x^2 + \frac{2M^2 \Phi_x \Phi_t}{V} + \frac{M^2 \Phi_t^2}{V^2} + \ldots \tag{23}$$

In rolling coordinates equation (23) becomes

$$C_p = -2\left(\Phi_x + \frac{pz}{V}\Phi_y - \frac{py}{V}\Phi_z + \frac{\Phi_t}{V}\right) - \Phi_y^2 - \Phi_z^2 + \beta^2 \Phi_x^2 + \frac{2M^2}{V}\Phi_x \Phi_t + \frac{M^2}{V^2}\Phi_t^2 + \ldots \tag{24}$$

For the flow associated with steady rolling equation (24) becomes (correct to the second order)

$$C_p = -2\left(\Phi_x + \frac{pz}{V}\Phi_y - \frac{py}{V}\Phi_z\right) - \Phi_y^2 - \Phi_z^2 + \beta^2\Phi_x^2 \tag{25}$$

Equation (25) will be used in the following analysis of rolling airfoils.

Method of Iteration:

It will be assumed that equation (18) can be solved by an iteration procedure. The first step is to find the first order solution. The first order partial differential equation is obtained by neglecting the second order terms in equation (18). Thus the first order differential equation is

$$-\beta^2 \phi_{xx} + \phi_{yy} + \phi_{zz} = 0 \tag{26}$$

The first order solution is taken as the first approximation to the solution of equation (18). It is assumed that the second approximation can be found by substituting the first order solution into the right side of equation (18) and solving the resulting nonhomogeneous equation, which from equations (18) and (26) is

$$-\beta^2 \psi_{xx} + \psi_{yy} + \psi_{zz} = -\frac{2Vp}{c^2}\left(y\,\phi_{xz} - z\,\phi_{xy}\right) +$$

$$2M^2\left[(N-1)\quad \beta^2 \phi_x\ \phi_{xx} + \phi_y\ \phi_{xy} + \phi_z\ \phi_{xz}\right] \tag{27}$$

where

$$N = \frac{(\gamma-1)\,M^2}{2\beta^2}$$

The solution of equation (27) will be referred to as the second order solution.

The particular integrals utilized in solving equation (27) will only apply to the problem under consideration. A different particular integral is required for each problem.

Boundary Conditions:

Physical considerations require that the flow be tangent to the surface, and that all velocity perturbations vanish upstream of the airfoil. These boundary conditions may be expressed mathematically as:

$\phi\,(x,y,z) = 0$ upstream of the airfoil,

and

$$q \cdot \nabla s = 0$$

where $s(x,y,z) = 0$ is the equation of the surface of the body.

The equation of the surface of the body may also be expressed as

$$z = \epsilon f(x,y)$$

where ϵ is small compared with unity and $f(x,y)$; thus,

$$\nabla s = -i\epsilon \frac{\partial f}{\partial x} - j\epsilon \frac{\partial f}{\partial y} + k$$

Since the velocity, q, can be written as

$$q = i\,(V + V\phi_x + V\psi_x) + j(V\phi_y + V\psi_y + pz) + k\,(V\phi_z + V\psi_z + py)$$

it follows that boundary condition on the body surface is given by

$$-(V + V\phi_x + V\psi_x)\,\sigma_x - (V\phi_y + V\psi_y + pz)\,\sigma_y + V\phi_z + V\psi_z + py = 0$$

where $\sigma_x = \epsilon \frac{\partial f}{\partial x}$ and $\sigma_y = \epsilon \frac{\partial f}{\partial y}$.

The coordinate axes will be chosen so that the airfoil under consideration lies approximately in the z = 0 plane. The boundary conditions for the first order solution are given by:

$$\phi\,(x,y,z) = 0 \text{ upstream of the airfoil,}$$

and

$$\phi_z \Big|_{z=0} = -\frac{py}{V} + \sigma_x \tag{28}$$

Similarly the boundary conditions for the second order solution are given by

$$\psi = 0 \text{ upstream of the airfoil}$$

and

$$\psi_z \Big|_{z=0} = \sigma_x \phi_x \Big|_{z=0} + \sigma_y \phi_y \Big|_{z=0} - \epsilon f(x,y)\,\phi_{zz} \Big|_{z=0} \tag{29}$$

Unfortunately for the airfoils considered in this paper the first order velocity components are discontinuous across the Mach sheet from the leading edge. The effect of these discontinuities on the second order solution must be evaluated.

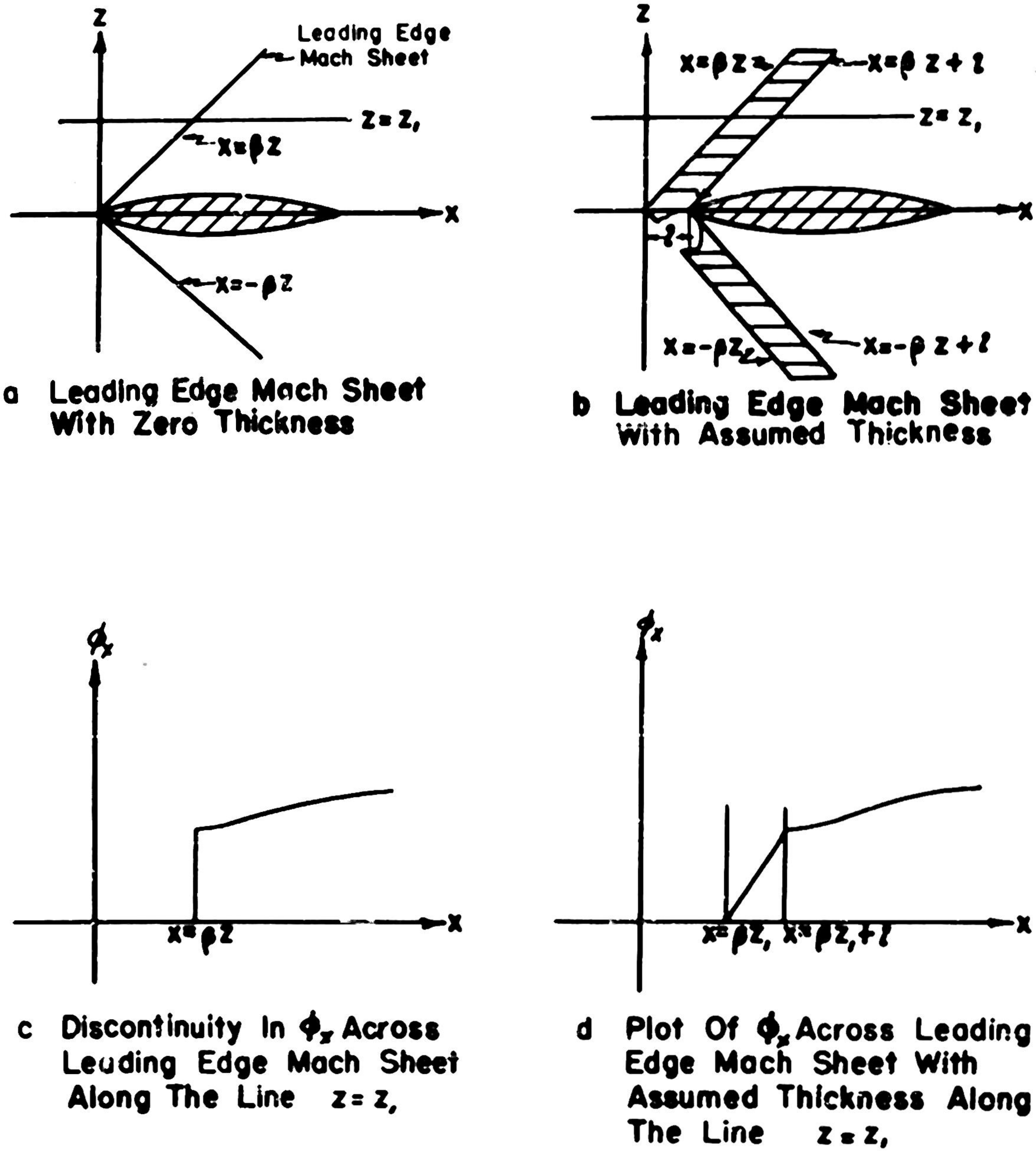


FIG. 1. An illustration of removing the discontinuity in ϕ_x across the leading edge Mach sheet by assuming the Mach sheet to have thickness.

In order to evaluate the effect of the discontinuities in the first order velocity components we shall assume that the leading edge Mach sheet has a small but finite thickness, within which the first order velocity components will be made continuous. Thus, the discontinuities through the Mach sheet are replaced by continuous functions. This process is illustrated in figure 1 for the velocity component in the free stream direction for an airfoil with leading edge perpendicular to the free stream. The effect of the discontinuities on the second order solution will be found by obtaining the second order solution within the Mach sheet and then letting the thickness approach zero.*

The procedure outlined in the preceding paragraph will be illustrated by considering the case of an infinite rolling rectangular airfoil having a constant initial slope, ϵ , along its leading edge. Only the Mach sheet above the airfoil will be considered since the Mach sheets above and below are of the same form. The discontinuities through the leading edge Mach sheet depend only on the initial slope of the airfoil; therefore the discontinuities in the first order velocity components through the Mach sheet above the airfoil will be the same as those through the Mach sheet above a flat rolling airfoil having a constant slope ϵ .

The first order potential function associated with the flow over the upper surface of an infinite rolling rectangular wing with a constant slope ϵ is given by

$$\phi = \left(\frac{py}{V} - \epsilon\right) \frac{(x-\beta z)}{\beta} \tag{30}$$

where the airfoil lies approximately in the $z=0$ plane**.

The discontinuities in the first order velocity components through the leading edge Mach sheet above the airfoil are from equation (30)

$$\Delta\phi_x = \frac{1}{\beta}\left(\frac{py}{V} - \epsilon\right)$$

$$\Delta\phi_y = 0$$

$$\Delta\phi_z = \left(\frac{-py}{V} + \epsilon\right)$$

It will be assumed that the Mach sheet has a small thickness (see Figure 1-b). The velocity components within the sheet will be defined as

---

* An alternate approach would be to attach a small surface to the leading edge of the airfoil which would cause the first order velocity components to be continuous. The effect of the discontinuities would then be evaluated by a limiting process in which the width of the small attached surface would approach zero.

** Equation (30) can be found by the integration of a source distribution; (see references 7 and 12) however, for simple problems such as this a heuristic method generally yields the result with a minimum of effort.

$$\phi_x = \left(\frac{py}{V} - \epsilon\right)\left(\frac{x - \beta z}{\ell \beta}\right)$$

$$\phi_y = 0 \qquad (31)$$

$$\phi_z = -\left(\frac{py}{V} - \epsilon\right)\frac{(x-\beta z)}{\ell}$$

where ℓ is the thickness of the Mach sheet in the x direction. Note that within the Mach sheet

$$-\beta^2 \phi_{xx} + \phi_{yy} + \phi_{zz} = 0$$

and that the velocity components are continuous functions in the neighborhood of it.

From equations (27) and (31) the differential equation within the Mach sheet is found to be

$$-\beta^2 \psi_{xx} + \psi_{yy} + \psi_{zz} = \frac{2M^2}{\ell}\left\{\frac{py}{V}\left(\frac{py}{V} - \epsilon\right) + \left[\frac{N\left(\frac{py}{V} - \epsilon\right)^2}{\ell}\right](x-\beta z)\right\} \qquad (32)$$

It is well known (references 7 and 13) that the solution of the nonhomogeneous equation

$$-\beta^2 \psi_{xx} + \psi_{yy} + \psi_{zz} = F(x,y,z)$$

is given by

$$\psi(x,y,z) = \frac{-1}{2\pi}\iiint_V \frac{F(\xi,\eta,\zeta)}{R}\, d\xi\, d\eta\, d\zeta + \frac{1}{2\pi}\int^f\int_s \left(\frac{1}{R}\nabla\phi - \phi\nabla\frac{1}{R}\right)\cdot \bar{n}_h \, da \qquad (33)$$

where

$$R = \sqrt{(x-\xi)^2 - \beta^2 (y-\eta)^2 - \beta^2 (z-\zeta)^2},$$

$\bar{n}_h = -i\beta^2 \nu_1 + j \nu_2 + k\nu_3$ (ν_1, ν_2, ν_3 are the direction cosines of the outward normal to the element of area da, s is an arbitrary surface which cuts the forward Mach cone from the point (x,y,z), v is the volume inclosed in the forward Mach cone from the point (x,y,z) and the surface s, and the symbol ${}^{f}\!\iint$ denotes the finite part of an integral as defined by Hadamard in reference (12).)

The result of applying equation (33) to equation (32) is

$$\psi(x,y,z) = -\frac{1}{2\pi}\iiint_v \frac{1}{R} \frac{2M^2}{\gamma}\left\{\frac{p\eta}{V}\left(\frac{p\eta}{V} - \epsilon\right) + \left[\frac{N\left(\frac{p\eta}{V} - \epsilon\right)^2}{\ell}\right](\xi - \beta\zeta)\right\} d\xi \, d\eta \, d\zeta + \tag{34}$$
$$\frac{1}{2\pi} {}^{f}\!\iint_s \left(\frac{1}{R}\nabla\phi - \phi\nabla\frac{1}{R}\right)\cdot \bar{n}_h \, da$$

where the volume, v, is the volume of the Mach sheet within the forward Mach cone of the point (x,y,z). The surface s will be chosen so that it is made up of the z=0 plane and an arbitrary surface upstream of the Mach sheet.

The integral over the surface upstream of the forward Mach sheet is zero so that the surface integral in equation (34) can be reduced to

$$\frac{1}{2\pi} {}^{f}\!\iint_s \left(\frac{1}{R}\nabla\phi - \phi\nabla\frac{1}{R}\right)\cdot \bar{n}_h \, da = -\frac{1}{2\pi} {}^{f}\!\iint_{s_1}\left[\frac{\phi_z(\xi,\eta,0)}{R} + \phi(\xi,\eta,0)\frac{\partial}{\partial z}\left(\frac{1}{R}\right)\right] d\xi \, d\eta$$

The area s_1 is the area of the z=o plane which is within the Mach sheet and the forward Mach cone from the point (x,y,z). From reference 7 page 22 the preceding integral can be expressed as

$$\frac{1}{2\pi} \int\int_{s}^{f} \left(\frac{1}{R} \nabla \phi - \phi \nabla \frac{1}{R}\right) \cdot \bar{n}_h \, da = -\frac{1}{\pi} \int\int_{s_1} \frac{\phi_z(\xi, \eta, 0)}{R} \, d\xi \, d\eta$$

It follows from this expression that equation (34) can be expressed as

$$\psi(x,y,z) = -\frac{M^2}{\pi \ell} \int\int\int_{V} \frac{1}{R} \left\{ \frac{p\eta}{V} \left(\frac{p\eta}{V} - \epsilon \right) + \frac{N \left(\frac{p\eta}{V} - \epsilon\right)^2}{\ell} (\xi - \beta \zeta) \right\} d\xi \, d\eta \, d\zeta + \tag{35}$$

$$+ \frac{1}{\pi \ell} \int\int_{s_1} \frac{\left(\frac{p\eta}{V} - \epsilon\right) \xi}{R} \, d\xi \, d\eta$$

In order to evaluate the second order solution on the down stream side of the Mach sheet the point (x,y,z) will be located at an arbitrary point on the surface $x = \beta z + \ell$. The thickness of the Mach sheet was chosen to be small. It follows that for given value of ξ and ζ the variations in the integrand of the volume and surface integrals are made up almost completely of the variation of $\frac{1}{R}$. This allows the substitution of $y = \eta$ in the integrands except in R. Equation (35) can now be reduced to

$$\psi(\beta z + \ell, y, z) = -\frac{M^2}{\pi \ell} \int_{0}^{z} \int_{\beta \zeta}^{\beta \zeta + \ell} \left\{ \frac{py}{V} \left(\frac{py}{V} - \epsilon \right) + \right.$$

$$+ \frac{N\left(\frac{py}{V} - \epsilon\right)^2}{\ell} \quad (\xi - \beta\zeta)\Bigg\}\, d\xi\, d\zeta \int_{y - \frac{\sqrt{(x-\xi)^2 + \beta^2(z-\zeta)^2}}{\beta}}^{y + \frac{\sqrt{(x-\xi)^2 + \beta^2(z-\zeta)^2}}{\beta}} \frac{d\eta}{R} \quad +$$

$$+ \frac{\left(\frac{py}{V} - \epsilon\right)}{\pi\ell} \int_0^{\ell} \xi\, d\xi \int_{y - \frac{\sqrt{(x-\xi)^2 + \beta^2(z-\zeta)^2}}{\beta}}^{y + \frac{\sqrt{(x-\xi)^2 + \beta^2(z-\zeta)^2}}{\beta}} \frac{d\eta}{R}$$

Integrating with respect to η in preceding expression gives

$$\psi(\beta z + \ell, y, z) = -\frac{M^2\pi}{\pi\ell\beta} \int_0^{z} d\zeta \int_{\beta\zeta}^{\beta\zeta+\ell} \Bigg\{ \frac{py}{V}\left(\frac{py}{V} - \epsilon\right) +$$

$$\frac{N\left(\frac{py}{V} - \epsilon\right)^2}{\ell} \quad (\xi - \beta\zeta)\Bigg\} \quad d\xi \quad + \frac{\left(\frac{py}{V} - \epsilon\right)}{\ell\beta} \int_0^{\ell} \xi\, d\xi$$

Performing the remaining integrations yields

$$\psi(\beta z + \ell, y, z) = -\frac{M^2 py}{\beta V} \left(\frac{py}{V} - \epsilon\right) z -$$

$$\frac{M^2}{2\beta}\left[z\, V^2 \quad N\left(\frac{py}{V} - \epsilon\right)^2\right] + \frac{\left(\frac{py}{V} - \epsilon\right)}{2\beta} \quad \ell$$

Taking the limit as ϵ approaches zero and replacing z by $\frac{x}{\beta}$ gives

$$\Delta \psi \Big|_{x = \beta z} = -\frac{M^2 (N+2)}{2\beta^2} \frac{p^2 y^2}{V^2} x + \frac{M^2 (N+1)}{\beta^2} \frac{p \epsilon yx}{V} + $$

$$- \frac{M^2 N \epsilon^2 x}{2\beta^2} \tag{36}$$

Equation (36) is the value of the discontinuity of the second order potential function associated with the flow over the upper surface of a rolling airfoil which has a leading edge with a constant initial slope and zero sweepback.

Similarly the discontinuity of the second order potential function associated with the flow over the upper surface of a rolling airfoil which has a sweptback leading edge with a constant initial slope is found to be

$$\Delta \psi \Big|_{mx-y-\alpha z=0} = \frac{M^2 m}{\alpha^2} \left\{ \frac{p^2}{3\alpha V^2} \left(\frac{\beta^2 m^2 N}{2\alpha^2} + 1 \right) \left(-3\alpha^2 y^2 + 3\alpha yz - z^2 \right) + \right.$$

$$\left. + \frac{p\epsilon}{2V} \left(\frac{\beta^2 m^2 N}{\alpha^2} + 1 \right) \left(2\alpha y - z \right) - \frac{N\beta^2 m^2 \epsilon^2}{2\alpha} \right\} z \tag{37}$$

where

$$\alpha = \sqrt{\beta^2 m^2 - 1}$$

The Infinite Rectangular Wing with an Arbitrary Cross Section:

In this section the second order potential function associated with an infinite rectangular wing of arbitrary cross section will be determined. The coordinate axes are chosen so that the y-axis coincides with the leading edge (see Figure 2). The flow over the upper and lower surfaces are similar; therefore, only the flow over the upper surface will be considered in detail. The equation of the upper surface will be represented by

$$z = \epsilon f(x) \tag{38}$$

where ϵ is small compared to unity and f (x) is of the order of unity.

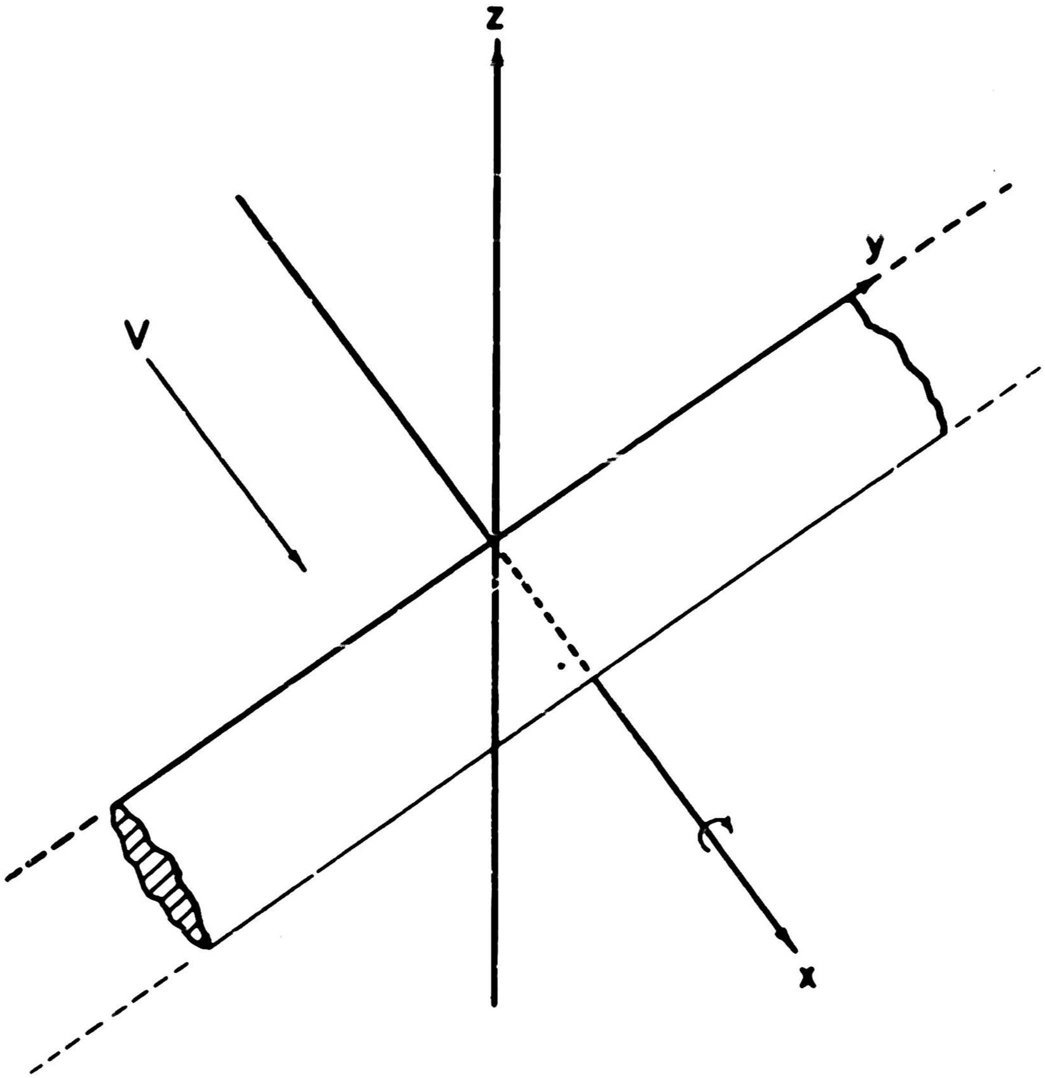


FIG. 2. Coordinate axes used in the analysis of an infinite rectangular wing.

By inspection

$$\Gamma_1 = \frac{py\epsilon}{V\beta^2}(M^2 N + \beta^2) f(x-\beta z)$$

$$\Gamma_2 = -\epsilon^2 \int_0^{x-\beta z} f(\xi)\, f''(\xi)\, d\xi$$

$$\Gamma_3 = -\frac{\epsilon^2 (M^2 N-2)}{2\beta^2} \int_0^{x-\beta z} \left[f'(\xi) \right]^2 d\xi$$

The function Γ_4 was found by assuming a solution of the form

$$\Gamma_4 = -\frac{M^2 (N+4)\, p^2 y^2 x}{2\beta^2 V^2} + K_1 x^3 + K_2 \left[(x+\beta z)^3 + (x-\beta z)^3 \right]$$

where K_1 and K_2 are undetermined constants. This leads directly to the result

$$\Gamma_4 = \frac{M^2 (N+4)\, p^2}{2\beta^2 V^2} = \left\{ -y^2 x - \frac{x^3}{3\beta^2} + \frac{1}{24\beta^2} \left[(x+\beta z)^3 + (x-\beta z)^3 \right] \right\}$$

Hence

$$\theta_2 = \sum_{i=1}^{4} \Gamma_i = \frac{py\epsilon}{V\beta^2} (M^2 N + \beta^2 - 1)\; f(x-\beta z) + \qquad (47)$$

$$-\epsilon^2 \int_0^{x-\beta z} f(z)\, f''(\xi)\, d\xi - \frac{\epsilon^2 (M^2N-2)}{2\beta^2} \int_0^{x-\beta z} \left[f'(\xi)\right]^2 d\xi + \tag{47}$$

$$+ \frac{M^2(N+4)p^2}{2\beta^2 V^2} \left\{ - y^2 x - \frac{x^3}{3\beta^2} + \frac{1}{24\beta^2} \left[(x+\beta z)^3 + (x-\beta z)^3 \right] \right\}$$

Since the second order potential function is expressed by

$$\Phi = \phi + \psi = \phi + \theta_1 + \theta_2$$

the second order potential function is given by (from equations (39), (42), and (47)).

$$\Phi = \frac{py}{V\beta}(x-\beta z) - \frac{\epsilon f(x-\beta z)}{\beta} + \frac{M^2 p^2 y^2}{\beta^2 V^2} x +$$

$$+ \frac{\epsilon\, py\, M^2(N+1)z}{V\beta} f'(x-\beta z) - \frac{\epsilon^2 M^2 Nz}{2\beta} \left[f'(x-\beta z)\right]^2 + \tag{48}$$

$$+ \frac{py\epsilon}{V\beta^2}(M^2N+\beta^2)\, f(x-\beta z) - \epsilon^2 f(x-\beta z)\, f'(x-\beta z) +$$

$$- \frac{\epsilon^2 M^2 (N-2)}{2\beta^2} \int_0^{x-\beta z} \left[f'(\xi)\right]^2 d\xi +$$

$$\frac{p^2 M^2}{2\beta^2 V^2}(N+4) \left\{ -y^2 x - \frac{x^3}{3\beta^2} + \frac{1}{24\beta^2} \left[(x+\beta z)^3 + (x-\beta z)^3 \right] \right\}$$

From equations (25) and (48) the pressure coefficient on the upper surface of the airfoil is given by

$$C_p = -2\left\{\frac{py}{V\beta} - \frac{\epsilon f'(x)}{\beta} + \frac{p\epsilon y M^2 N}{V\beta^2} f'(x) - \frac{\epsilon^2 (M^2N-2)}{2\beta^2}\left[f'(x)\right]^2 + \right.$$

$$\left. - \frac{p^2 M^2}{2\beta^2 V^2}(N+4)\left(y^2 + \frac{3x^2}{4\beta^2}\right)\right\} \tag{49}$$

For airfoils of symmetrical thickness distribution at zero angle of attack the pressure coefficient on the lower surface of the airfoil is the same as that on the upper surface with p replaced by -p. The pressure difference coefficient can therefore be expressed as

$$\Delta C_p = C_p(p) - C_p(-p)$$

It follows from the preceding relation and equation (49) that

$$\Delta C_p = -\frac{4py}{V\beta}\left[1 + \frac{\epsilon}{\beta} M^2 N f'(x)\right] \tag{50}$$

Note that the second order effect in equation (50) is a function only of the local slope of the surface, $\epsilon f'(x)$.

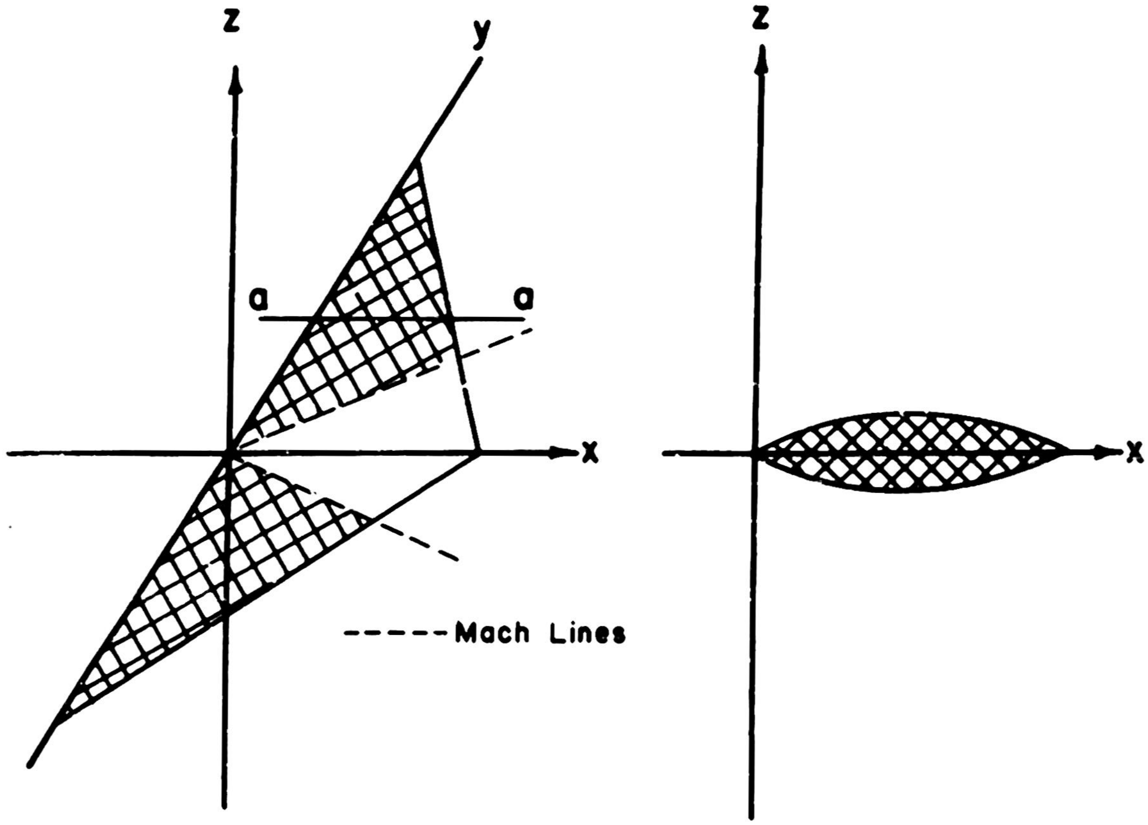
It should be noted that only terms multiplied by p and $p\epsilon$ contribute to the lifting pressure. This is true in general since the nonhomogeneous equation used in finding the second order solution is linear and each term on the right side of the equation is multiplied by either p, ϵ, p^2, $p\epsilon$, or ϵ^2. In the following analysis terms not multiplied by p or $p\epsilon$ will therefore be neglected.

A Region of Flow Over a Reversed Delta Wing:

The potential function associated with the region of flow which is not affected by the center section of a reversed delta wing will be found for a wing cross section given by

$$z = \epsilon x\left(\frac{\lambda d - 2y}{\lambda} - x\right) \tag{51}$$

The region of flow considered and the cross section are illustrated in figure 3.



a. Cross Hatched Region Indicates The Region Of Flow Not Affected By The Center Section

b. Cross Section View In Plane a–a

FIG. 3. The reversed delta wing with parabolic cross section.

$$-\beta^2 \frac{\partial^2\theta_2}{\partial x^2} + \frac{\partial^2\theta_2}{\partial y^2} + \frac{\partial^2\theta_2}{\partial z^2} = 0$$

$$\theta_2 \Big|_{x=\beta z} = 0 \quad ; \quad \frac{\partial\theta_2}{\partial z}\Big|_{z=0} = \frac{\partial\psi}{\partial z}\Big|_{z=0} ,$$

and

$$-\beta^2 \frac{\partial^2\theta_3}{\partial x^2} + \frac{\partial^2\theta_3}{\partial y^2} + \frac{\partial^2\theta_3}{\partial z^2} = 0$$

$$\theta_3 \Big|_{x=\beta z} = \psi\Big|_{x=\beta z} \quad ; \quad \frac{\partial\theta_3}{\partial z}\Big|_{z=0} = 0.$$

It is easy to verify that the functions θ_1, θ_2 and θ_3 are

$$\theta_1 = -\frac{M^2 p\epsilon\,(N+1)\,(x^2 - \beta^2 z^2)}{\beta^2 V}\, y + \tag{a}$$

$$+ \frac{8M^2 p\epsilon}{V\beta^2\lambda}\left[\frac{xz^2}{2} - \frac{(x+\beta z)^3}{16\beta^2} - \frac{(x-\beta z)^3}{16\beta^2}\right] +$$

$$-\frac{2M^2 p\epsilon}{3\lambda\beta V}\left[z^3 - \left(\frac{x+\beta z}{2\beta}\right)^3 - \left(\frac{x-\beta z}{2\beta}\right)^3\right]$$

$$\theta_2 = -\frac{p\epsilon\, dy}{\beta^2 V}(x-\beta z) + \frac{p\epsilon\, y}{\beta^2 V}(x-\beta z)^2 + \frac{2p\epsilon\,(x-\beta z)^3}{3\lambda\beta^2 V} + \tag{56}$$

$$+\frac{p\epsilon}{\lambda\beta^2 V}\left(\frac{x^2}{3\beta^2} - \frac{2z^2}{3} + 2y^2 + \frac{xz}{3\beta}\right)(x-\beta z) \tag{b}$$

and

$$\theta_3 = -\frac{M^2 p\epsilon (N+1)}{\beta^2 V}\left[\frac{2y^2x}{\lambda} + \frac{2x^3}{3\lambda\beta^2} - \frac{(x+\beta z)^3}{12\lambda\beta^2} + \right. \quad \text{(c)}$$

$$\left. - \frac{(x-\beta z)^3}{12\lambda\beta^2} - dyx\right]$$

The second order solution is

$$\Phi = \phi + \psi = \phi + \theta_1 + \theta_2 + \theta_3$$

or from equations (52) and (56) (neglecting terms not multiplied by p or $p\epsilon$)

$$\Phi = \frac{py}{V\beta}(x-\beta z) - \frac{M^2 p\epsilon (N+1)(x^2-\beta^2 z^2)y}{\beta^2 V} +$$

$$+ 4\frac{M^2 p\epsilon}{V\beta^2\lambda}\left[xz^2 + \frac{(x+\beta z)^3}{8\beta^2} - \frac{(x-\beta z)^3}{8\beta^2}\right] - \frac{p\epsilon dy}{\beta^2 V}(x-\beta z) +$$

$$+ \frac{p\epsilon y(x-\beta z)^2}{\beta^2 V} + \frac{2p\epsilon (x-\beta z)^3}{3\lambda\beta^2 V} + \frac{p\epsilon}{\lambda\beta^2 V}\left(\frac{x^2}{3\beta^2} + \right. \quad (57)$$

$$\left. - \frac{2z^2}{3} + 2y^2 + \frac{xz}{3\beta}\right)(x-\beta z) +$$

$$- \frac{M^2 p\epsilon (N+1)}{\beta^2 V}\left[\frac{2y^2x}{\lambda} + \frac{2x^3}{3\lambda\beta^2} - \frac{(x+\beta z)^3}{12\lambda\beta^2} - \frac{(x-\beta z)^3}{12\lambda\beta^2} - dyx\right] +$$

$$- \frac{2M^2 p\epsilon}{3\lambda\beta V}\left[z^3 - \left(\frac{x+\beta z}{2\beta}\right)^3 - \left(\frac{x-\beta z}{2\beta}\right)^3\right]$$

From equations (25) and (57) the pressure difference coefficient is given by

$$\Delta C_p = -4\left\{\frac{py}{V\beta} - \frac{3p}{2\beta^4 V\lambda}(M^2N + 2)x^2 + \frac{p\epsilon M^2 N}{\beta^2 V}dy + \right.$$

$$\left. - \frac{p\epsilon 2M^2 Nxy}{\beta^2 V} - \frac{p\epsilon 2 M^2 Ny^2}{\beta^2 V\lambda}\right\} \quad (58)$$

The chord-wise lifting pressure is plotted in Figure (4) for a reversed delta wing with a 5 per cent thick center section at a Mach number of 2 for two spanwise stations. The chordwise lifting pressure variations presented in figure (4) are quite similar to those for infinite wings with the same cross sections.

The Infinite Sweptback Rectangular Wing With an Arbitrary Cross Section:

In this section the effect of sweepback will be investigated by considering the infinite sweptback rectangular wing with an arbitrary cross section. The coordinate axes are chosen as indicated in figure (5). The equation of the upper surface will be represented by

$$z = \epsilon f\left(\frac{mx-y}{m}\right) \tag{59}$$

The first order solution is

$$\phi = \frac{p}{2V\alpha^3}(mx-y-\alpha z)\left[-mx + (2\beta^2 m^2 - 1)y - \alpha z\right] + \tag{60}$$

$$-\frac{\epsilon m}{\alpha} \quad f\left(\frac{mx-y-\alpha z}{m}\right)$$

The second order potential function must satisfy the nonhomogeneous equation (neglecting terms multiplied by p^2 and ϵ^2)

$$-\beta^2 \psi_{xx} + \psi_{yy} + \psi_{zz} = \frac{2M^2 p\epsilon}{V}\left\{-\left(\frac{N\beta^4 m^4}{\alpha^4} + 1\right) y f'' + \frac{z}{\alpha} f'' + \right.$$

$$\left. + \frac{\beta^2 m^3 N}{\alpha^4} f' + \frac{N\beta^2 m^3}{\alpha^4} x f''\right\}$$

The function ψ will be divided into two parts θ_1 and θ_2. The function θ_1 is taken to be a solution of the nonhomogeneous equation, therefore θ_2 is a solution of the homogeneous equation.
The function θ_1 was chosen to be

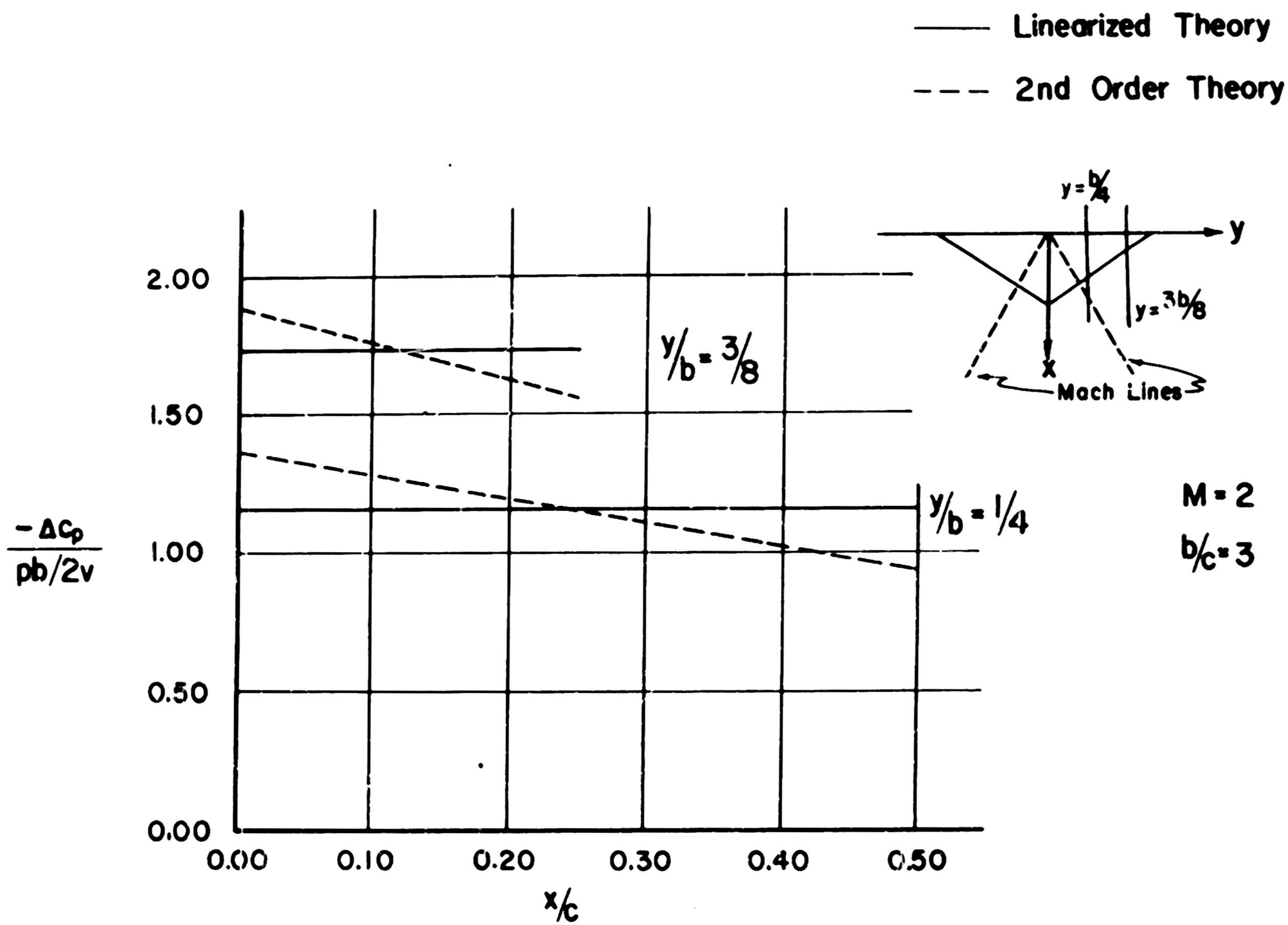


FIG. 4. Distribution of the chordwise lifting pressure coefficient, $-2V\Delta C_p/pb$, on a reversed delta wing with a 5% thick center section with a parabolic profile, at Mach number 2.0.

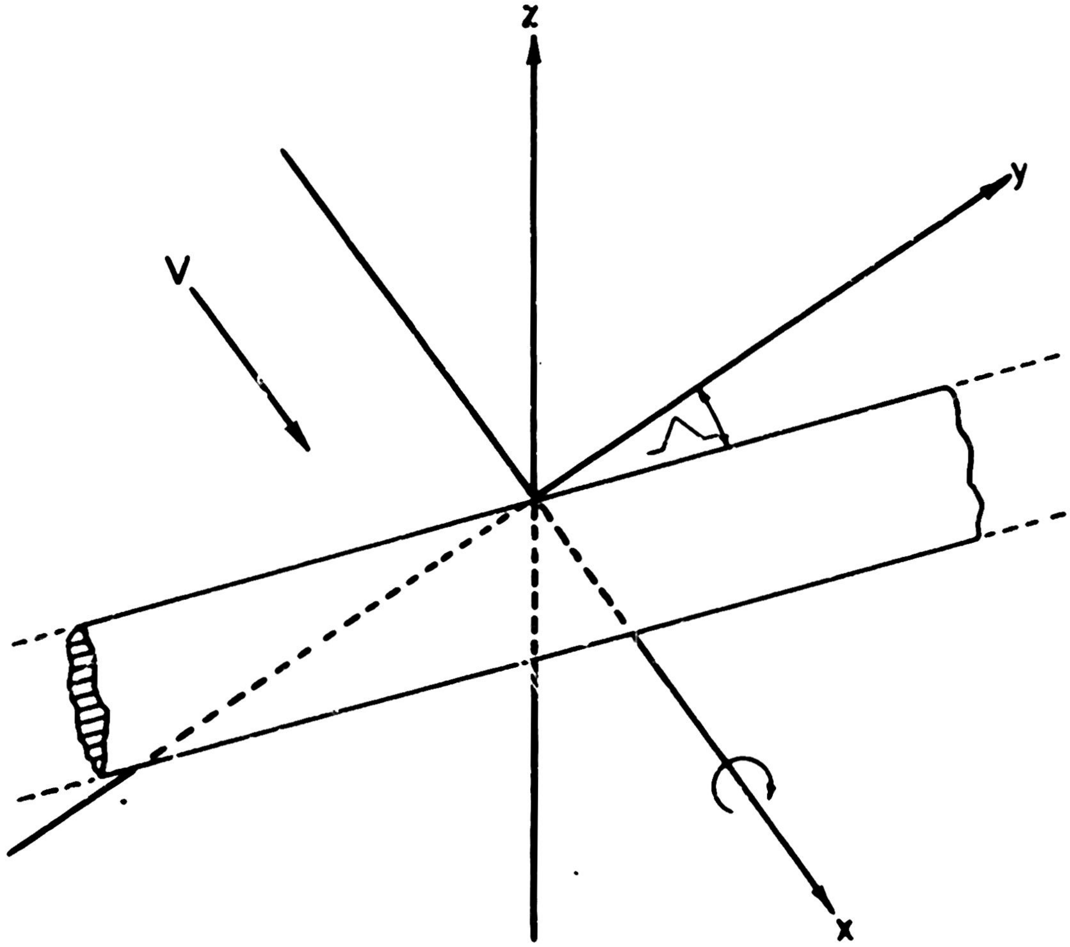


FIG. 5. Coordinates for sweptback wing.

$$\Delta C_p \Big|_{\text{surface}} = \frac{4pm}{V\alpha^3} (mx - \beta^2 m^2 y) - \frac{4p\epsilon M^2 m^2}{V \alpha^4} \Bigg\{ \frac{1}{2} \left(\frac{N\beta^2 m^2}{\alpha^2} + 1 \right) \cdot$$

$$\cdot (mx-y) f'(0) + \left(\frac{N\beta^4 m^4}{\alpha^2} - 1 \right) y f' + \tag{67}$$

$$- \left(\frac{N\beta^2 m^2}{\alpha^2} - 1 \right) mxf' - m \left(\frac{2N\beta^2 m^2}{\alpha^2} + 1 \right) f \Bigg\}$$

Figure (6) presents the variation of the chordwise lifting pressure on a wedge with a 45° sweepback and with a 10 per cent thickness at a Mach number of $\sqrt{5}$.

THE DAMPING IN ROLL

The pressure coefficients found in the preceding sections can be used to investigate the effect of thickness on the damping in roll of some airfoils. In the following sections some exact and approximate expressions are presented for the damping in roll of several types of plan forms.

The Infinite Rectangular Wing:

The damping in roll coefficient, C_{l_p}, can be written as

$$C_{l_p} = \lim_{p \to 0} \frac{1}{\frac{pb}{2V} Sb} \iint_{\substack{\text{wing} \\ \text{area}}} y \, \Delta C_p \, dx \, dy \tag{68}$$

For an infinite rectangular wing the C_{l_p} can be written as

$$C_{l_p} = \lim_{\substack{b \to \infty \\ p \to 0}} \frac{1}{\frac{pb}{2V} Sb} \int_{-b/2}^{b/2} \int_{0}^{d} y \, \Delta C_p \, dx \, dy \tag{69}$$

Substituting equation (50) into equation (69) and performing the indicated mathematical operations yields

$$C_{l_p} = - \frac{2}{3\beta} \left[1 + \frac{\epsilon}{\beta} M^2 N \frac{f(d)}{d} \right] \tag{70}$$

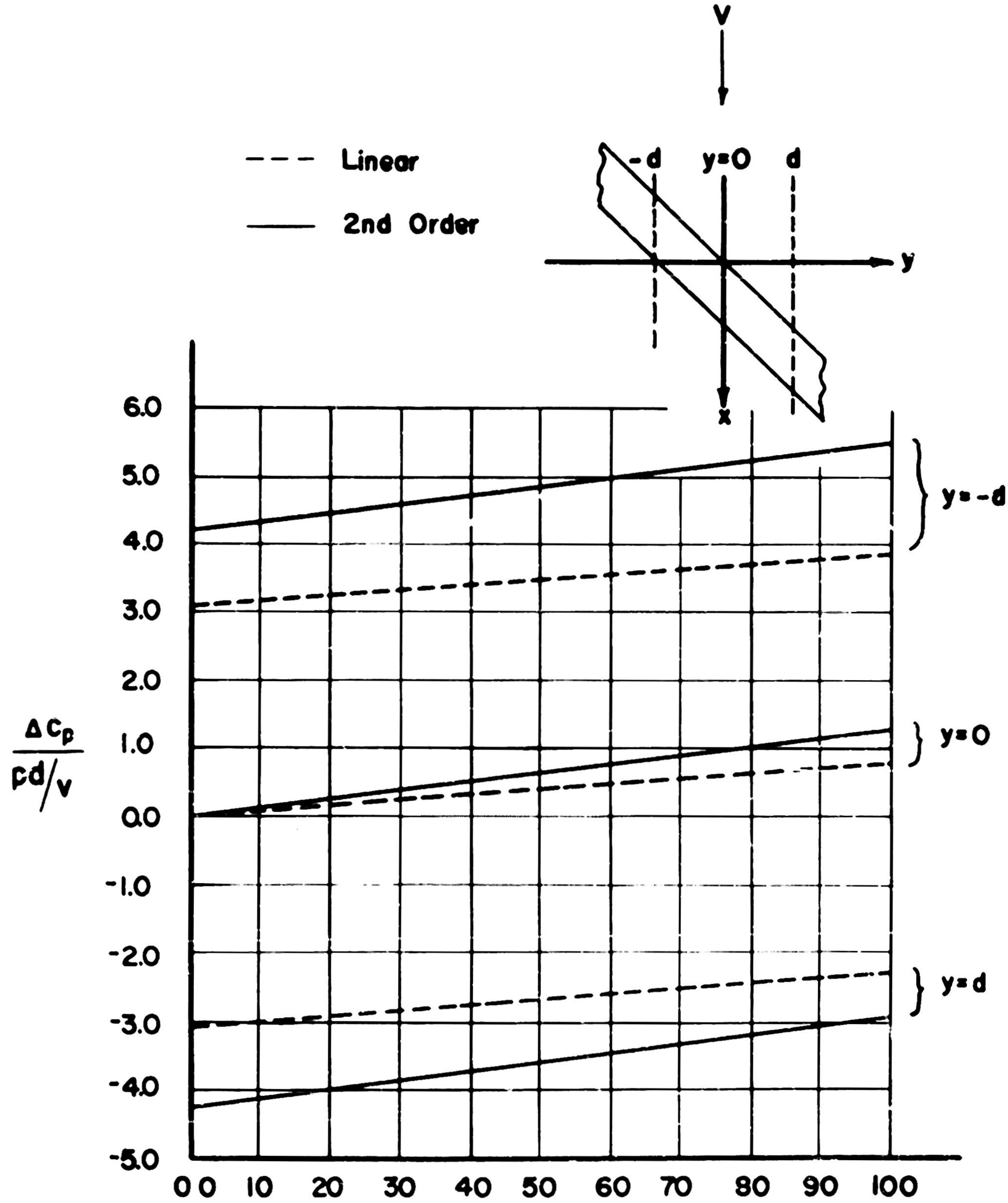


FIG. 6. The chordwise variation of the lifting pressure $-V\Delta C_p/\rho d$, on a wedge with a 45° sweep-back and a 10% thickness at Mach number $\sqrt{5}$.

Equation (70) indicates that the effect of thickness on the C_{l_p} of an infinite rectangular airfoil with a symmetrical thickness distribution varies directly as the thickness of the trailing edge. It can also be shown that to the second order the C_{l_p} is independent of the angle of attack of the airfoil.

From equation (50) the center of pressure on one panel (half-wing) is found to be

$$\bar{x} = d - \frac{d/2 + \frac{\epsilon N M^2}{\beta d} \int_0^d f(x)\,dx}{1 + \frac{\epsilon}{\beta} M^2 N \frac{f(d)}{d}} \qquad (a)$$

(71)

$$\bar{y} = b/3 \qquad (b)$$

where $\bar{x}$ is measured from the airfoil's leading edge and $\bar{y}$ is measured from the airfoil's center line. For the infinite wedge the preceding equations reduce to

$$\bar{x} = d/2$$
$$\bar{y} = b/3$$

Figure (7) presents the variation of $\bar{x}/d$ with Mach number for various values of the thickness parameter for an infinite wing with a parabolic cross section.

The Reversed Delta Wing:

An approximate expression for the damping in roll of the reversed delta wing previously considered can be obtained by assuming the expression for the pressure distribution in the region not affected by the center section is valid over the complete wing.

Under this assumption the (approximate) C_{l_p} of the reversed delta wing is given by (from equations (58) and (69)).

$$C_{l_p} = -\frac{1}{6\beta} + \frac{2\epsilon d(M^2 N + 2)}{5\beta^4 \; A\!\!R^2} \qquad (72)$$

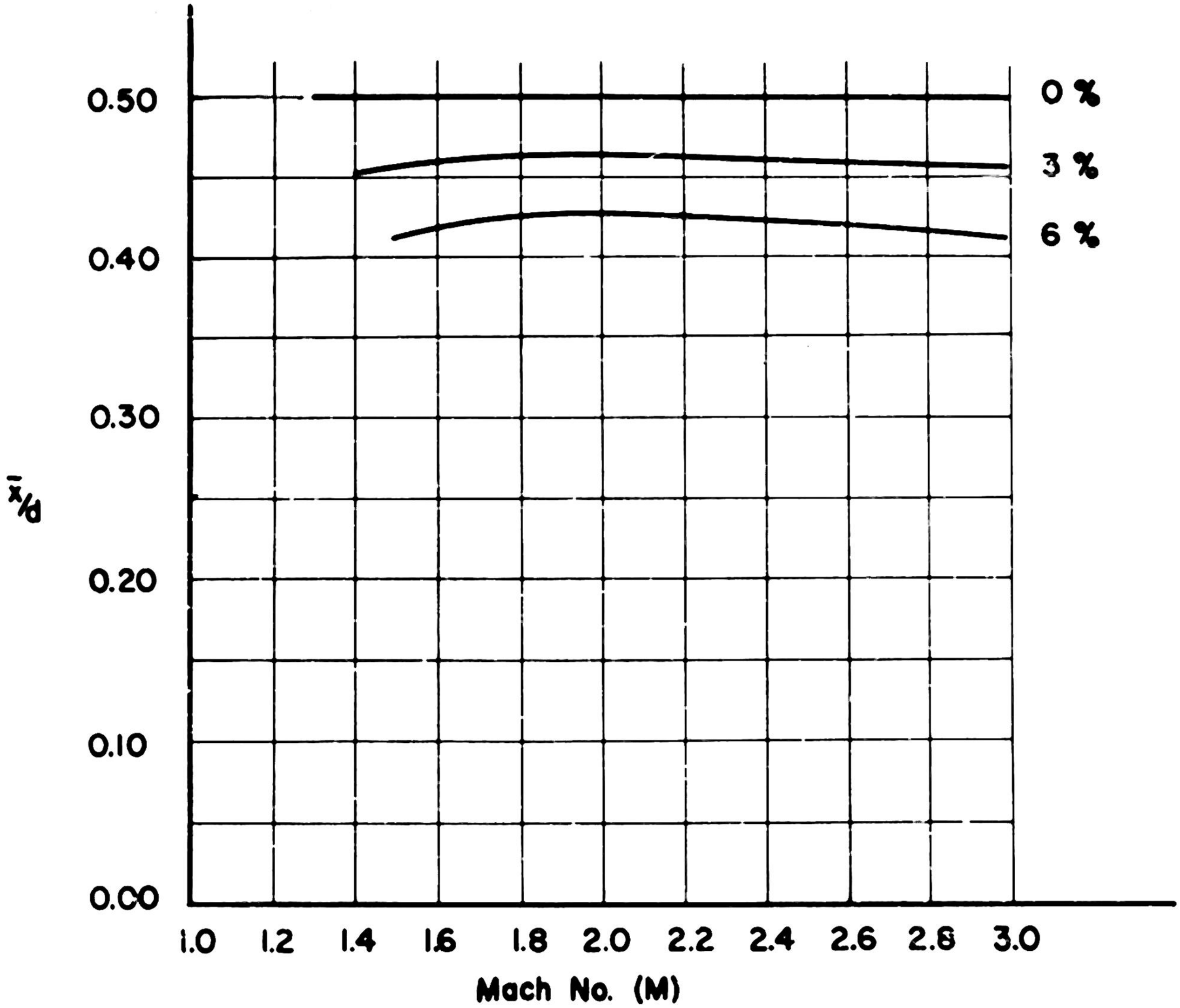


FIG. 7. The variation of the chordwise location of center of pressure, $\bar{x}/d$, with Mach number for various values of the thickness parameter for an infinite wing with a parabolic cross-section.

Figure (8) presents the variation of the C_{l_p} with Mach number for 0 and 5 per cent thick center sections for various aspect ratios. It can be seen from Figure (8) that the effect of thickness is quite small in this case.

From equation (58) the center of pressure is located approximately at the point

$$\bar{x} = d\left[\frac{\frac{1}{12} - \frac{6\epsilon d(M^2N+2)}{5\beta^3 AR^2} - \frac{M^2N\epsilon d}{60\beta}}{\frac{1}{3} - \frac{2\epsilon d(M^2N+2)}{\beta^3 AR^2}}\right] \quad (a)$$

$$(73)$$

$$\bar{y} = b\left[\frac{\frac{1}{12} - \frac{\epsilon d(M^2N+2)}{10\beta^3 AR}}{\frac{1}{3} - \frac{2\epsilon d(M^2N+2)}{\beta^3 AR^2}}\right] \quad (b)$$

Figures (9) and (10) present the variation of $\bar{x}/d$ and $\bar{y}/b$ respectively with Mach number for various aspect ratios for reversed delta wings with 0 and 5 per cent thick center sections.

The Infinite Swept Wing:

The damping in roll of an infinite swept wing with an arbitrary symmetrical cross section can be obtained by substituting equation (67) into equation (68) and performing the indicated mathematical operations. The C_{l_p} for this type of airfoil is given by

$$C_{l_p} = -\frac{2m}{3a}\left[1 + \frac{\epsilon M^2 N\beta^2 m^3 f(d)}{a^3 \quad d}\right] \quad (74)$$

Equation (74) indicates that the effect of thickness on the C_{l_p} of an infinite swept wing with an arbitrary symmetrical thickness distribution varies directly as the thickness of the trailing edge.

Figure (11) presents the variation of the C_{l_p} with Mach number for 10 per cent infinite wedges for various sweepback angles.

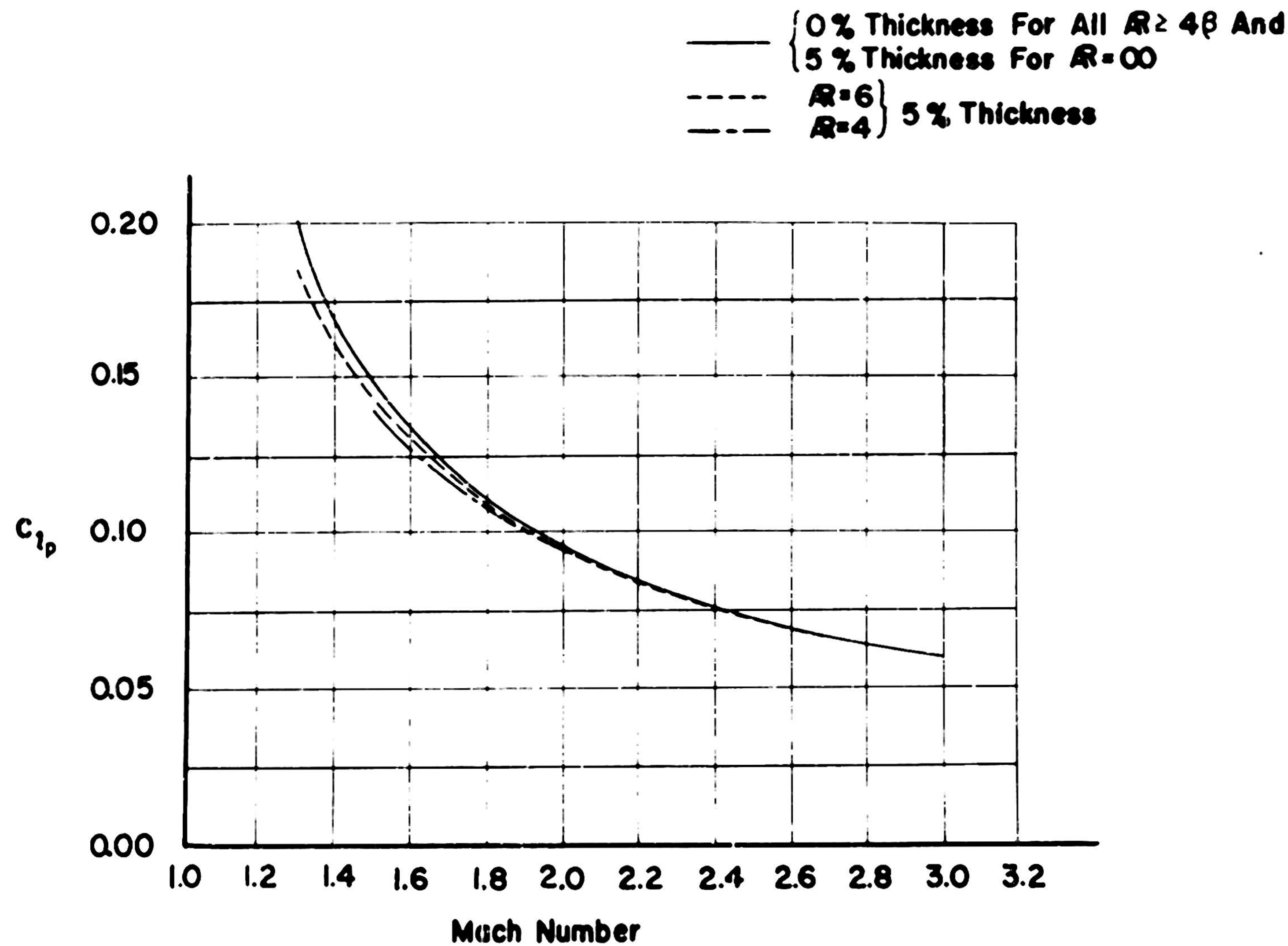


FIG. 8. The variation of the (approximate) damping in roll coefficient, C_{l_p}, with Mach number for zero and five per cent thick center sections for a reversed delta wing with parabolic center sections.

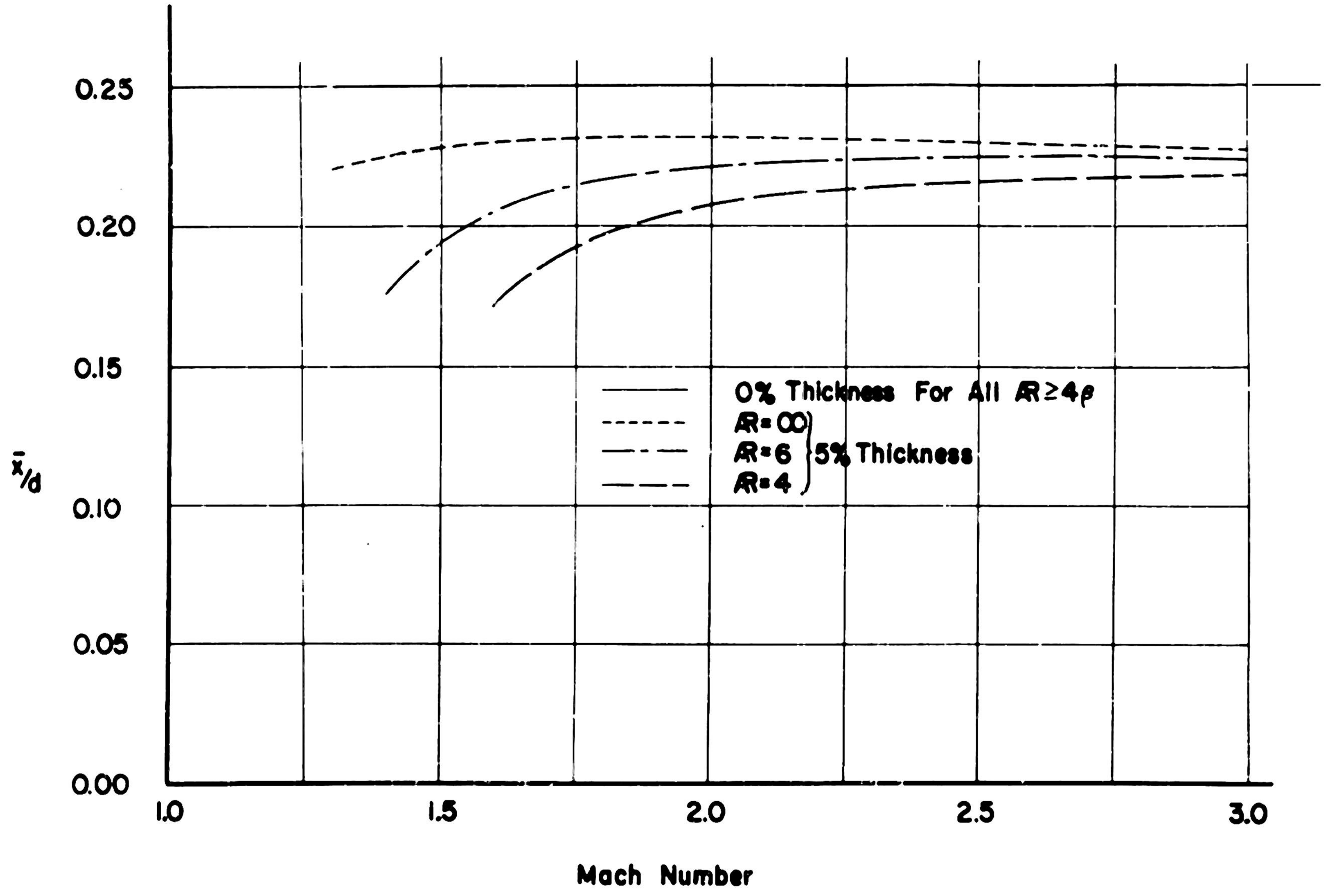


FIG. 9. The variation of the chordwise location of center of pressure, $\bar{x}/d$, with Mach number for various aspect ratios for reversed delta wings of parabolic cross section with zero and 5% thick center sections.

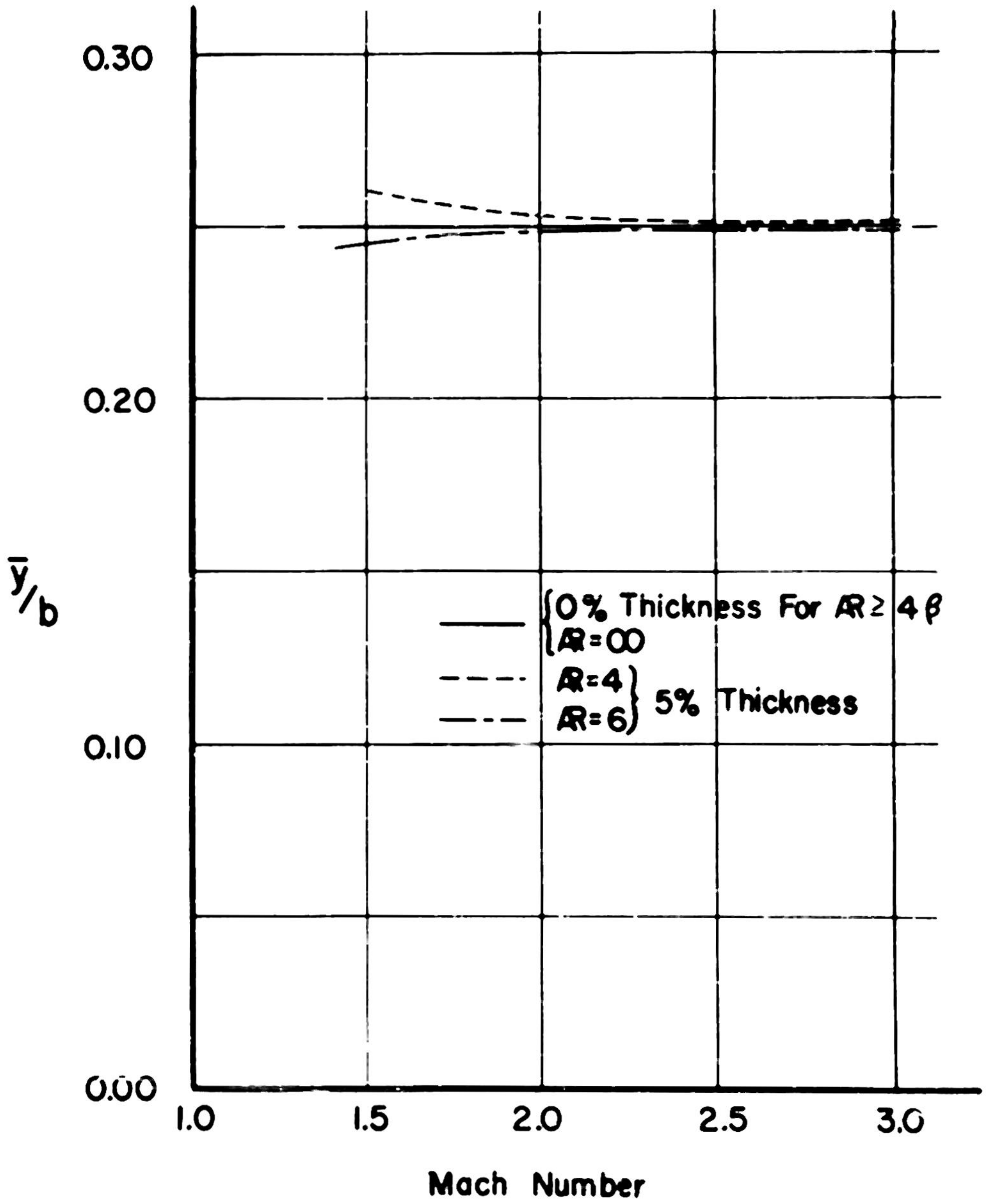


FIG. 10. The variation of the spanwise location of center of pressure, $\bar{y}/b$, with Mach number for various aspect ratios for reversed delta wings of parabolic cross section with zero and 5 per cent thick center sections.

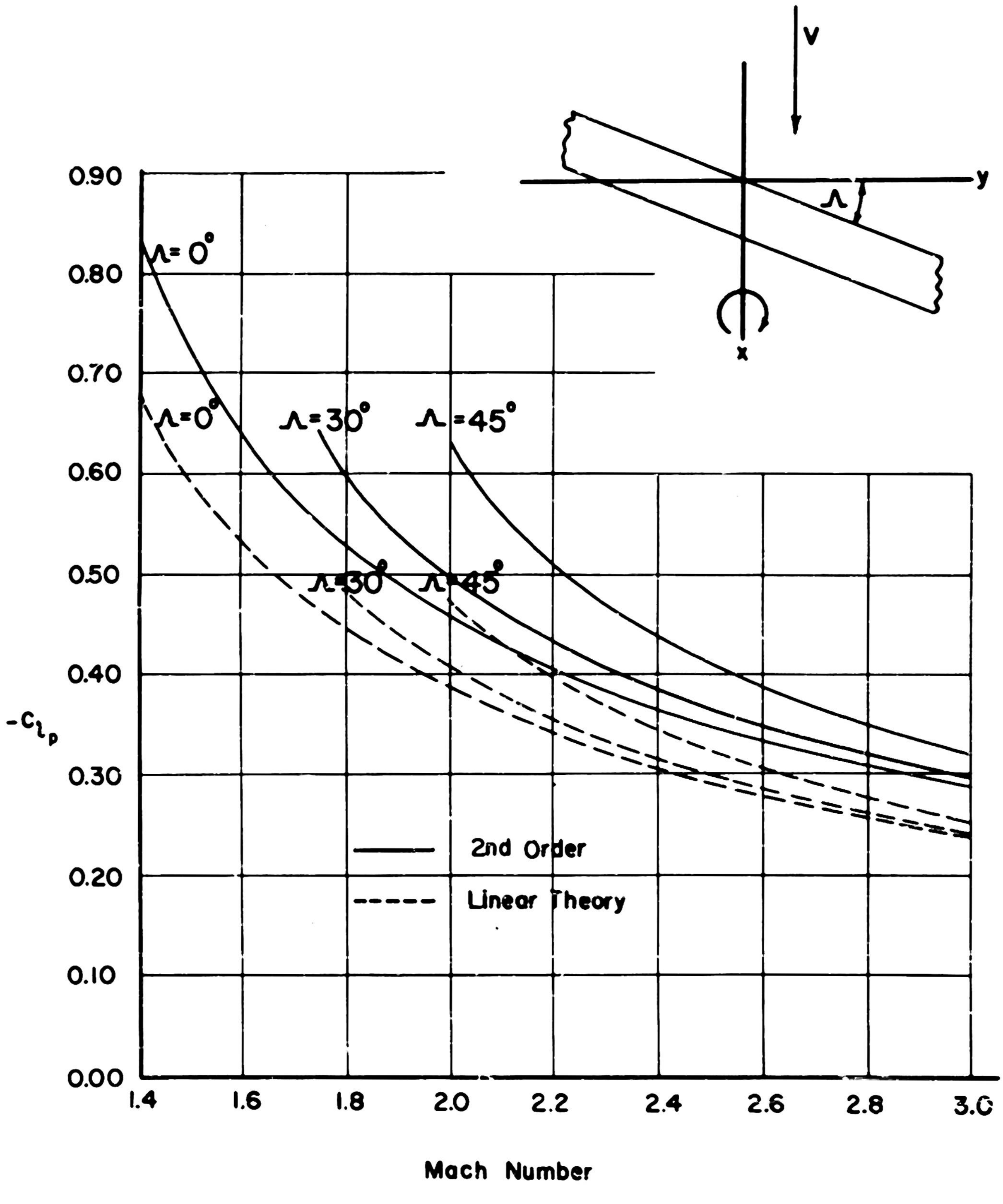


FIG. 11. The variation of the damping in roll coefficient, C_{l_p}, with Mach number for 10% thick infinite wedges for various sweepback angles.

The Swept Fin:

Figure 6 indicates that spanwise loading on an infinite swept wing is not symmetrical. An approximate expression for the C_{l_p} of a swept fin will therefore be presented. It is assumed that pressure distribution is the same as that for an infinite swept wing and that the roll axis is located as indicated in Figure 12.

The C_{l_p} of a fin may be expressed as

$$C_{l_p} = \frac{1}{\frac{pb}{V} S b} \int_0^b \int_{y/m}^{(y/m)+d} y \Delta C_p \, da \tag{75}$$

Substituting equation (67) into equation (75) and performing the indicated mathematical operations yields

$$C_{l_p} = -\frac{4m}{3\alpha}\left[1 + \frac{\epsilon M^2 N \beta^2 m^3}{\alpha^3}\left(\frac{f_{TE}}{d}\right) + \frac{m^2}{\alpha^3 AR} + \right. \tag{76}$$

$$+ \frac{\epsilon}{AR}\left\{ -\frac{M^2 m^3}{2\alpha^4}\left(\frac{N\beta^2 m^2}{\alpha^2} + 1\right) f'(0) + \frac{2M^2 m^3}{\alpha^4}\left(\frac{N\beta^2 m^2}{\alpha^2} - 1\right)\left(\frac{f_{TE}}{d}\right) + \right.$$

$$\left. + \frac{4M^2 m^3}{\alpha^4}\left(\frac{N\beta^2 m^2}{2\alpha^2} + 1\right) \int_0^d \frac{f(u)}{d^2} \, du \right\}$$

Figure 13 presents the variations of the C_{l_p} with aspect ratio for a 10 per cent thick wedge for various sweepback angles at Mach number $\sqrt{5}$. This figure indicates that for a fin with a wedge cross section the difference between the C_{l_p} of the linearized theory and the C_{l_p} of the second order theory increases with sweepback angle.

Figure 14 presents the variations of the C_{l_p} with aspect ratio for a fin with a cross section at $y = 0$ (5 per cent thick) given by

$$z = \frac{x(d-x)}{10d}$$

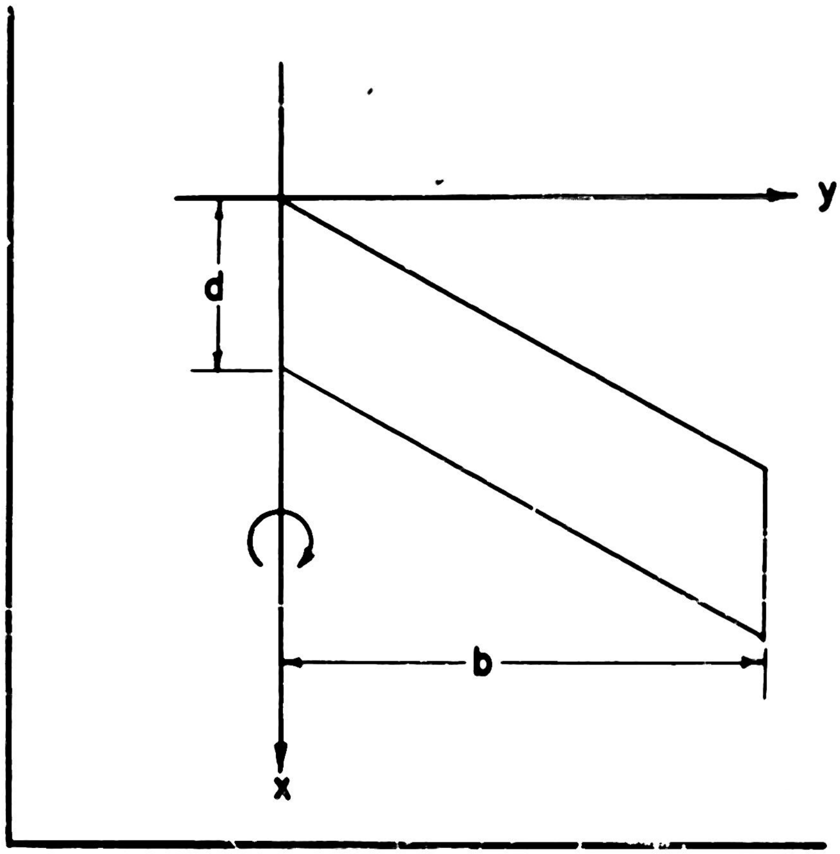


FIG. 12. Coordinates used in finding the approximate damping in roll coefficient, C_{l_p}, for a fin.

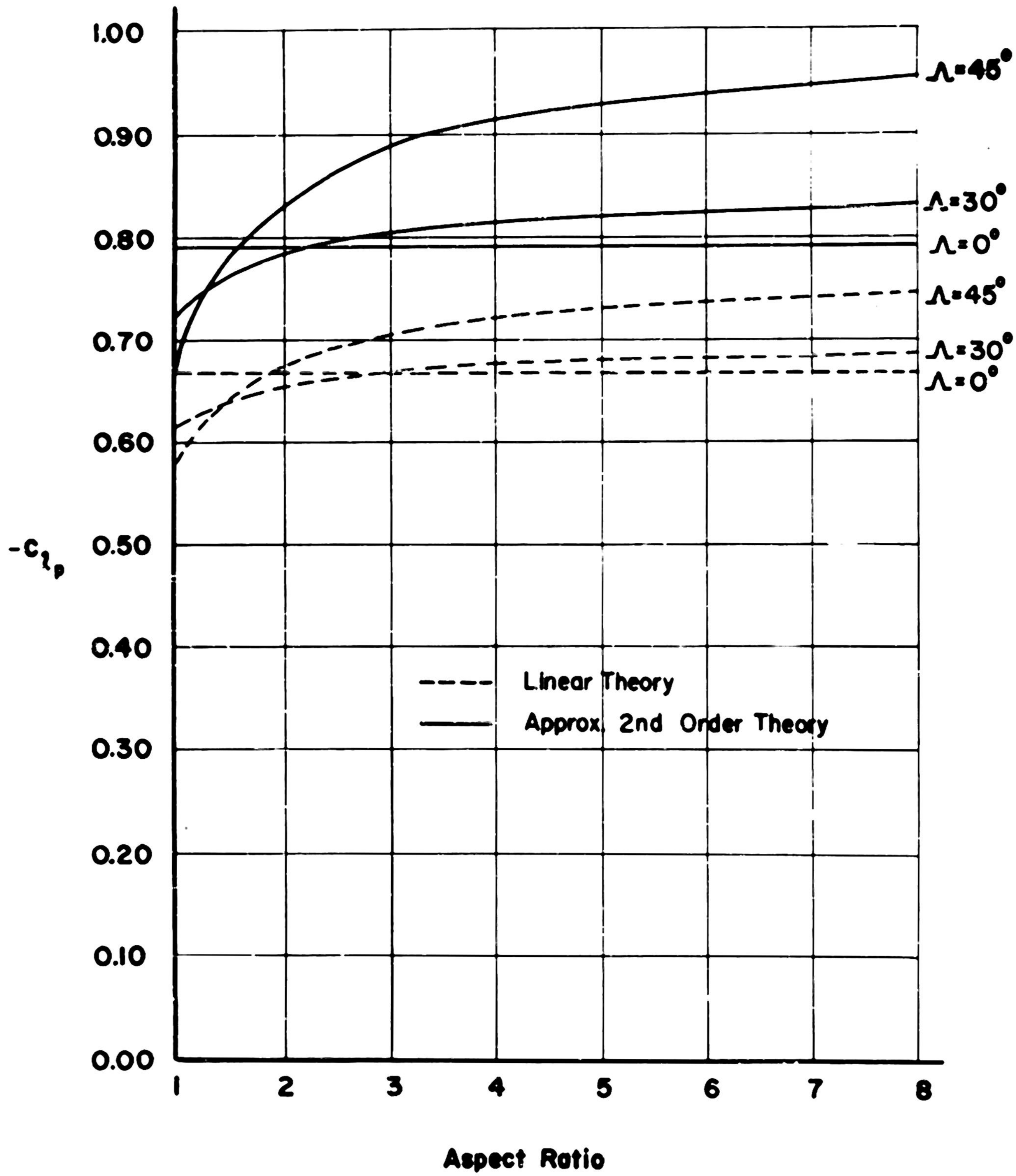


FIG. 13. The variation of the approximate damping in roll coefficient, C_{l_p}, with aspect ratio for a 10% thick fin with a wedge cross section for various sweepback angles at Mach number 2.236.

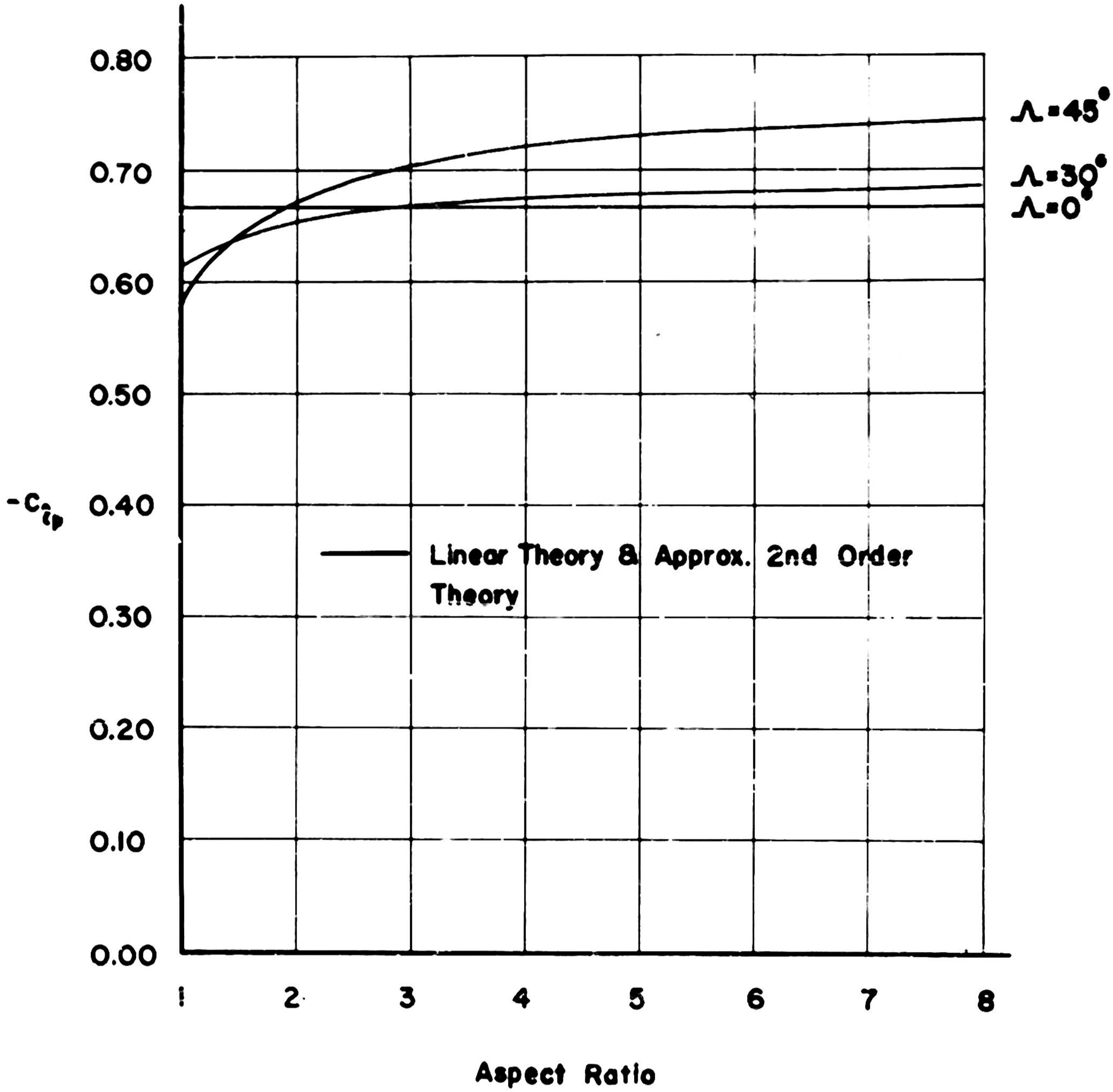


FIG. 14. The variation of the approximate damping in roll coefficient, C_{ℓ_p}, with aspect ratio for a 5% thick fin with a parabolic cross section for various sweepback angles at Mach number 2.236.

for various sweepback angles at Mach number $\sqrt{5}$. This figure indicates that for a fin with a parabolic cross section there is little difference between the values of the linear theory C_{l_p} and those of the second order theory C_{l_p}.

CONCLUDING REMARKS

Most of the airfoils considered in the preceding analysis are not generally encountered in practice; however, many airfoils do have regions with the same pressure distribution as airfoils considered here. It is, therefore, possible to estimate fairly accurately the effect of thickness on the C_{l_p} of many airfoils not considered in the analysis. As an illustration consider the C_{l_p} of a rectangular airfoil with a wedge cross section such as the rectangular fins considered in reference 14. Figure 8 of reference 14 presents the C_{l_p} of rectangular fins with 8 per cent thickness. An estimate of the effect of thickness can be made by increasing the C_{l_p} of the linearized theory by the same percentage as the thickness increases the C_{l_p} for an infinite wing. Figure 15 presents the values of the C_{l_p} estimated in this manner as compared to the experimental values presented in reference 14. It can be seen from this figure that the agreement between theory and experiment has been improved considerably.

The results of the present paper seem to indicate two general trends:

1. The effect of thickness on the C_{l_p} is quite small for the airfoils with zero thickness trailing edges.

2. The effect of thickness on the C_{l_p} increases slightly with sweepback angle.

The airfoils considered in this paper have symmetrical thickness distributions. But since the flows over the upper and lower surfaces of the airfoils treated herein are independent of each other, aerodynamic properties of airfoils with similar planforms but with unsymmetrical thickness distributions can easily be determined from the results contained here.

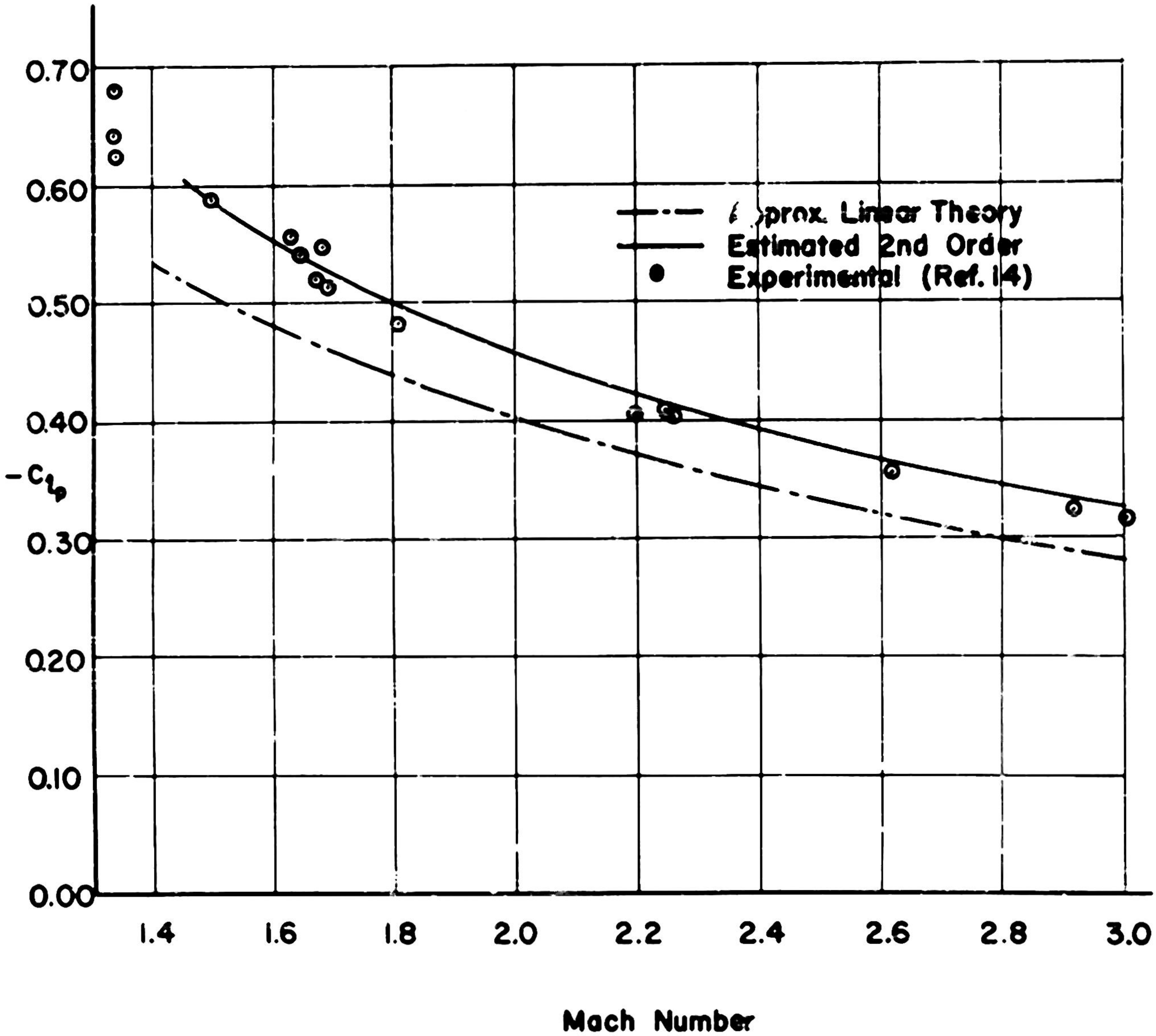


FIG. 15. A comparison of the estimated second order damping in roll coefficient, C_{l_p}, with experimental values for an 8% thick wedge.

The limitations of the Busemann second order theory for two dimensional flows have been investigated (see reference 15). Since the theory contained in the present paper is closely associated with the Busemann second order theory it seems likely that the results presented herein have similar limitations.

The type of analysis used in the present paper is not restricted to the study of airfoils with a constant rate of roll. An examination of the partial differential equations and of the boundary conditions involved indicates that the same type of analysis can be applied to airfoils at a constant angle of attack, airfoils with a constant rate of pitch, and airfoils with simple unsteady motions.

JOHN C MARTIN

John C. Martin

NATHAN GERBER

Nathan Gerber

REFERENCES

1. Harmon, S. M.: Stability Derivatives of Thin Rectangular Wings at Supersonic Speeds, N.A.C.A. TN 1706, Nov. 1948.

2. Malvestuto, F. S., Jr., Margolis, K., and Ribner, H. S.: Theoretical Lift and Damping in Roll at Supersonic Speeds of Thin Sweptback Wings with Streamwise Tips, Subsonic Leading Edges, and Supersonic Trailing Edges. NACA Rep. 970, 1950.

3. Piland, Robert O.: Summary of the Theoretical Lift, Damping-in-Roll, and Center-of-Pressure Characteristics of Various Wing Plan Forms at Supersonic Speeds. NACA TN 1977, 1949.

4. Bleviss, Zegmund O.: Some Roll Characteristics of Cruciform Delta Wings at Supersonic Speeds. Preprint of paper presented at Annual Summer Meeting, Inst. Aero. Sci. (Los Angeles), July 12, 1950.

5. Ribner, Herbert S.: Damping in Roll of Cruciform and Some Related Delta Wings at Supersonic Speeds. NACA TN 2285, 1951.

6. Harmon, S. M., and Jeffreys, J.: Theoretical Lift and Damping in Roll of Thin Wings with Arbitrary Sweep and Taper at Supersonic Speeds. Supersonic Leading and Trailing Edges. NACA TN 2114, 1950.

7. Martin, John C.: A Vector Study of Linearized Supersonic Flow-Applications to Nonplanar Problems. NACA TN 2641, 1952.

8. Bolz, R. E. and Nicolaides, J. D.: A Method of Determining Some Aerodynamic Coefficients from Supersonic Free-Flight Tests of a Rolling Missile. Jour. Aero. Sci., Vol. 17, No. 10, Oct. 1950, pp. 609-621.

9. Busemann, A.: Aerodynamischer Auftrieb bei Überschallgeschwendigkeit. Atti die convegni 5 R. Accad. d'Italia, 1936, pp. 328-360. (also printed in Luftfahrtforschung Bd. 12, Nr. 6, Oct. 3, 1935, pp. 210-229; available in translation as British Air Ministry Translation No. 2844.)

10. Van Dyke, M. D.: A Study of Second-Order Supersonic-Flow Theory. NACA TN 2200, 1951.

11. Van Dyke, M. D.: First and Second-Order Theory of Supersonic Flow Past Bodies of Revolution. Jour. Aero. Sci., Vol. 18, No. 3, March 1951, pp. 161-179.

12. Puckett, Allen E.: Supersonic Wave Drag of Thin Airfoils. Jour. Aero. Sci., Vol. 13, No. 9, Sept. 1946, pp. 475-484.

13. Hadamard, Jacques; Lectures on Cauchy's Problem in Linear Partial Differential Equations. Yale Univ. Press (New Haven), 1923.

14. Nicolaides, J. D. and Bolz, R. E.: On the Pure Rolling Motion of Winged/or Finned Missiles in Varying Supersonic Flight. Ballistic Research Laboratories (Aberdeen Proving Ground, Md.) Report 799.

15. Hilton, W. F.: High-Speed Aerodynamics. Longmans, Green and Co. (New York) 1951, pp. 237-240.